SINGLE-POINT METHOD OF IDENTIFICATION OF MAGNETO-HYDRODYNAMIC DISCONTINUITIES

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(Received 2 June, 1978)

Abstract. For each type of discontinuity the magneto-hydrodynamic relationships are reduced to a set of necessary relations between the parameters at both sides of the shock. The relations which do not contain the pressure are used in the method proposed. The distance in the space of the parameters between the point corresponding to the experimental data and the nearest point exactly satisfying the relations is obtained for each case. Supposition is made that the shortest distance conforms to the most probable type of discontinuity.

As an illustration of the proposed method several events of discontinuities experimentally registered in the interplanetary space are considered.

A number of methods are known for identification and determination of the characteristics of experimentally discovered discontinuities in the interplanetary plasma with the data related to the surroundings of a single point of the discontinuity. The best grounded, from the gas dynamics point of view, of such methods appears to be that described by Chao (1970) and Lepping and Argentiero (1971). The substance of such methods is that parameters up to and after the discontinuity, varying within experimental limits, satisfy the Rankine–Hugoniot relations with maximum likelihood in the magneto-gas dynamic approximation (relations not containing a pressure term). By a modification of the parameters the normal and velocity of propagation of the shock wave may be obtained. It is clear, however, that a selection of cases registered experimentally which 'resemble' shock fronts must first be brought together and the appropriate details evaluated. The absence of a strict determining criterion for such a selection introduces a certain unsatisfactory feature into the situation.

In consequence of the fact that errors of measurement in cosmic experiments are very large, it is possible, as was shown by Baranov and Kartalev (1974), by varying the values of parameters within the limits of such errors, to satisfy relationships not only for shock fronts but other types of discontinuity. In the present work an attempt is made to give an objective identification criterion for discontinuities, based on the ideas contained in the aforementioned papers.

It is supposed that the experimental data do not refer to the pressure and the Rankine–Hugoniot relationship is considered only for isotropic magnetic gas dynamics in the absence of pressure.

As is well known, different forms of discontinuity may be characterized in the following way (Landau and Lifschitz, 1970; Kulikovski and Lyubimov, 1962). Let, in

Astrophysics and Space Science **62** (1979) 29–33. 0004-640X/79/0621-0029$00.75

what follows, $\mathbf{v}$ denote the velocity, $\mathbf{B}$, the magnetic induction vector, and ϱ, the density. Then we have:

(1) Tangential discontinuity:

$$(\mathbf{v}_2 - \mathbf{v}_1)(\mathbf{B}_2 \times \mathbf{B}_1) = 0. \tag{1}$$

The normal to the surface of the discontinuity, and the velocity of its propagation D are given by the equations

$$\mathbf{n} = \frac{\mathbf{B}_1 \times \mathbf{B}_2}{|\mathbf{B}_1 \times \mathbf{B}_2|}, \qquad D = \mathbf{v}_1\mathbf{n}. \tag{2}$$

(2) Contact discontinuity:

$$\mathbf{B}_2 - \mathbf{B}_1 = 0, \qquad \mathbf{v}_2 - \mathbf{v}_1 = 0. \tag{3}$$

The normal vector $\mathbf{n}$ and the velocity D cannot be determined in this case.

(3) Perpendicular shock front:

$$B_n = 0, \qquad m_1 = m_2 \neq 0, \qquad m = \varrho(v_n - D)$$
$$\mathbf{v}_{\tau 2} - \mathbf{v}_{\tau 1} = 0, \qquad \frac{\mathbf{B}_2}{\varrho_2} - \frac{\mathbf{B}_1}{\varrho_1} = 0. \tag{4}$$

where the index n and τ designate the projections, respectively, in the normal and tangential planes to the shock front. For $\mathbf{n}$ and D we have

$$\mathbf{n} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{|\mathbf{v}_2 - \mathbf{v}_1|}, \qquad D = \mathbf{n}\,\frac{\mathbf{v}_2\varrho_2 - \mathbf{v}_1\varrho_1}{\varrho_2 - \varrho_1}. \tag{5}$$

(4) Inclined shock front:

$$B_{n1} = B_{n2}, \qquad m_1 = m_2,$$
$$m(\mathbf{v}_{\tau 2} - \mathbf{v}_{\tau 1}) = \frac{B_n}{4\pi}(\mathbf{B}_{\tau 2} - \mathbf{B}_{\tau 1}) \tag{6}$$
$$B_n(\mathbf{v}_2 - \mathbf{v}_1) = m\left(\frac{\mathbf{B}_2}{\varrho_2} - \frac{\mathbf{B}_1}{\varrho_1}\right)$$
$$\mathbf{n} = \frac{(\mathbf{B}_2 - \mathbf{B}_1) \times (\mathbf{B}_2 \times \mathbf{B}_1)}{|(\mathbf{B}_2 - \mathbf{B}_1) \times (\mathbf{B}_2 \times \mathbf{B}_1)|}, \qquad D = \mathbf{n}\,\frac{\mathbf{v}_2\varrho_2 - \mathbf{v}_1\varrho_1}{\varrho_2 - \varrho_1}. \tag{7}$$

(5) Rotational (Alfvén) discontinuity:

$$\varrho_2 - \varrho_1 = 0, \qquad B_2^2 - B_1^2 = 0, \qquad v_2^2 - v_1^2 = 0. \tag{8}$$

The normal vector may be obtained by solving the system

$$(\mathbf{B}_2 - \mathbf{B}_1)\mathbf{n} = 0; \qquad (\mathbf{v}_2 - \mathbf{v}_1)\mathbf{n} = 0; \qquad n^2 = 1. \tag{9}$$

The velocity of propagation D is determined from the expression

$$D = \mathbf{v}_1\mathbf{n} - \frac{\mathbf{B}_1\mathbf{n}}{\sqrt{(4\pi\varrho)}}. \tag{10}$$

Let us consider the point X with coordinates X_i $(i = 1, 2, \ldots, 14)$ in the 14 dimensional space of dimensionless parameters. The forms of the parameters $B_{j1,2}$, $v_{1,2}$, $\varrho_{1,2}$ $(j = 1, 2, 3)$ are known. We designate by X^0 the point corresponding to the experimentally obtained values, then we consider for each of the five types of discontinuity the sum

$$\sigma^2 = \sum_{i=1}^{14} (X_i - X_i^0)^2. \tag{11}$$

We seek to minimize this sum under conditions corresponding to the fulfilling of the requisite relationships. The problem of a constrained extremum may be solved numerically by the method of sequential unconstrained minimization (the method of reducing functions); of Fiocco and McCormick (1968). For each type of discontinuity we find $\sigma^2_{\min}$ at the corresponding point X^*. Using X^* we may find $\mathbf{n}$ and D for each type of discontinuity (excluding contact).

This supposes that the least scattering of $\sigma^2_{\min}$ will correspond to the most probable type of discontinuity. Since the relations (1) can be satisfied for any kind of discontinuity, it is clear that $\sigma^2_{\min}$ will always be least for the tangential discontinuity. Hence, in order to clarify whether the discontinuity is tangential or another type, it is necessary to secure additional information.

An EVM program was constructed, and the preceding method was applied to an amount of experimental data published in the literature. In particular, we have evaluated the data on seven cases of shock waves published by Chao (1970). As may have been expected, in each of these cases the most probable type of discontinuity is a tangential one or a shock wave; and, moreover, in the cases of shock waves the parameters published by Chao (1970) and those obtained by us after the appropriate minimization technique differ by not more than a few percent.

For the sake of illustration, let us quote the results for one of the seven cases given by Chao (1970), obtained on 29th August, 1966. Data from the experiment (in RTN coordinates) for this case are

TABLE I

	B_R	B_T	B_N	v_R	v_T	v_N	ϱ
Pre-Shock	−2.8	0.3	−2.4	355	−22	19	4.7
Post-Shock	−3.9	2.7	−8.2	343	8	8	14.5

where the magnetic field is given in gammas, the velocity in kilometers per second, and the density in cm^{-3}.

For the different types of discontinuity we obtain the following values for the parameters, corresponding **n** and D and the comparison parameter σ^2_{min} after carrying out the constrained minimization of σ^2:

Tangential discontinuity:

−2.8	0.3	−2.4	354.9	−21.9	19.0	4.7
−3.9	2.7	−8.2	343.0	7.9	8.0	14.5

(The parameters are hereafter listed in the same order as in Table I.)

Normal vector: 0.26, −0.87, −0.41
Velocity of the discontinuity: 103
Comparison parameter: 0.000 17

The value of the normal vector is obtained everywhere with regard to the sign.

Contact discontinuity:

−3.4	1.5	−5.3	394	−7	13.6	4.7
−3.4	1.5	−5.3	394	−7	13.3	14.5

Comparison parameter: 0.482

The normal vector and the propagation velocity are not defined here.

Perpendicular shock front:

−2.1	0.9	−0.8	405	−43	37	4.8
−6.4	2.7	−2.4	387	28	−9	14.5

Normal vector: 0.20, −0.82, 0.53
Propagation velocity: 7.3
Comparison parameter: 0.6787

Shock front:

−2.8	0.3	−2.4	355	−22	19	4.7
−3.9	2.7	−8.2	343	8	8	14.5

If we interpret the results with greater accuracy by a tangential discontinuity, there is a certain variation of the initial values of quantities.

Normal vector: 0.95, 0.31, −0.05, (0.97, 0.25, −0.04)
Propagation velocity: 455, (468)
Comparison parameter: 0.000 18

The values of the normal vector and propagation velocity obtained by Chao (1970) are given in brackets.

Rotational discontinuity:

−5.0	0.5	−4.3	394	−24	21	9.6
2.7	1.9	−5.7	395	7	7	9.6

Normal vector: 0.33, 0.36, 0.87
Velocity: 158
Comparison parameter: 1.537

Of course, particular interest attaches to those cases in which the proposed identification according to the method here presented gives a result differing from that given previously. As a qualitative illustration of such a case, we may consider the discontinuity registered by Marine V (day 179), which according to Turner (1973), was identified as a rotational discontinuity. In the same paper data are presented which support such a conclusion. However, application of our method shows that the value for the comparison parameter in the case of the rotational discontinuity and the shock front are, respectively, 5, 5×10^{-2} and 8.9×10^{-4}. Consequently, we must suppose that the shock wave is the most probable type unless there are additional reasons for identifying the discontinuity with the tangential type, for which the comparison parameter is 2.7×10^{-5}.

Acknowledgment

The authors wish to thank V. B. Baranov for useful discussions.

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