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On the correct posing of the method of transfinite interpolation for solving free-boundary problems of fluid flows

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Introduction

In recent years the fluid flows with free or moving boundaries enjoy a considerable interest both from theoretical and experimental point of view. This is due to their importance in a number of applications, e. g. motions of drops and bubbles through liquids, cavitation, waves on fluid surfaces, etc. At the same time, on the way of theoretical modelling of free-surface flows lie formidable obstacles founded in the strong non-linearity of the respective boundary value problems. That is why the only way to obtain theoretical predictions is the numerical one, except for certain limiting cases for which asymptotic analytical methods can be developed.

The most natural way to construct a numerical algorithm is to introduce new independent variables in a way to get the region of flow fixed and stationary in terms of the new coordinates. Moreover, the moving boundaries become coordinate lines. This approach is so attractive that in last two decades an extensive literature has been published concerning the construction of adaptive meshes (see for detailed survey [1]). Most of these papers, however, are concerned with constructing so-called body-fitted coordinate meshes for bodies with intricated but still fixed shape.

When the boundary of the flow is not priory known, but rather is governed by an equation the complexity of the problem increases significantly since the correctness of such an equation may depend on the way in which the grid is constructed. That is why the advanced grid generation techniques are not yet commonly used in numerical treatment of flows with moving boundaries.

The simplest and most obvious way to obtain boundary-fitted coordinate system is to scale one of the coordinates by the shape function of the boundary. In this case one arrives to well posed coordinate transformation only if the shape function is a single-valued function of the respective coordinate. Regardless to its limitations the method of scaled variable has been successfully applied in [2], [3] to the problem of bubble motion in ideal liquid and in [4] — in viscous liquids. This method proves efficient also for treating the cavitating flows (see [5]). It turned out to be very helpfull in investigation of the global correctness of viscous flows with free boundaries [6].

It seems reasonable to try to overcome the restriction of single-valued boundary shape function without sacrificing the simplicity and numerical effectiveness. The present paper is managed to show how a method for algebraic grid generation introduced by Coons (see [7]) and named recently "transfinite interpolation" (see [8], [9]) can be applied to flows with free boundaries. In order to display the mathematical problems which arise on this way the problem of ideal flow around gas bubble is considered.

1. Ideal flow around gas bubble

Consider a gas bubble occupying the region D of the three-dimensional space. Assume that the flow around bubble is axisymmetric and potential. Then it is governed by the two-dimensional Laplace equation:

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi}{\partial r}\right)+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\varphi}{\partial\theta}\right)=0,\quad (r,\ \theta)\in R^3\setminus D, \tag{1.1}$$

where r, θ are spherical coordinates (see Fig. 1) In the terms of spherical coordinates the region of the flow transforms to a region with two straight boundaries and one unknown curvilinear boundary. The latter corresponds to bubble surface (see Fig. 2). The fourth boundary is at infinity, but we can think that it is set at $r=R_k$ since the infinite regions cannot be treated numerically.

Boundaty conditions are the symmetry conditions

$$\frac{\partial\varphi}{\partial\theta}=0 \quad \text{for} \quad \theta=0,\ \pi, \tag{1.2}$$

the condition at infinity

$$\varphi\to U_\infty R_k\cos\theta \quad \text{for} \quad r=R_k, \tag{1.3}$$

and conditions on the moving bubble surface. Let the latter be represented by the following function

$$\varphi(r,\ \theta,\ t)=0. \tag{1.4}$$

Then the kinematic condition is

$$\frac{\partial\Phi}{\partial t}+\frac{\partial\varphi}{\partial r}\frac{\partial\Phi}{\partial r}+\frac{1}{r^2}\frac{\partial\varphi}{\partial\theta}\frac{\partial\Phi}{\partial\theta}=0 \quad \text{for} \quad \Phi(r,\ \theta,\ t)=0, \tag{1.5}$$

while the Cauchy-Lagrange integral (dynamic condition) reads:

$$\frac{\partial\varphi}{\partial t}+\frac{1}{2}\left[\left(\frac{\partial\varphi}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial\varphi}{\partial\theta}\right)^2\right]+\frac{p}{\rho}-gr\cos\theta+\sigma\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$
$$=\frac{1}{2}U_\infty^2+\frac{1}{\rho}p_\infty=F(t) \quad \text{for} \quad \Phi(r,\ \theta,\ t)=0, \tag{1.6}$$

where σ is surface tension, R_1, R_2 — the two principal radii of curvature of surface, p — pressure, g — gravity acceleration, p_∞ — pressure at infinity.

The principle trait of boundary value problem (1.1)—(1.6) is that one of the equations (1.5) or (1.6) is in fact an implicit equation for defining the shape functio (1.4). This makes the boundary value problem under consideration highly intractabe numerically even though leaving apart the fact that the unknown boundary is no coordinate line.

2. Coordinate transformation

Following the main idea of linear transfinite interpolation [8], [9] we introduce the following coordinate transformation

(2.1) $$r=\eta R_k+(1-\eta)R(\xi,\ t) \quad \text{and} \quad \theta=\eta\pi\xi+(1-\eta)G(\xi,\ t),$$

where

(2.2) $$r=R(\xi,\ t) \quad \text{and} \quad \theta=G(\xi,\ t)$$

is the parametric representation of curve (1.4). On function $G(\xi,\ t)$ are imposed the constraints

(2.3) $$G(0,\ t)=0, \quad G(1,\ t)=\pi\xi$$

which are required in order to secure the angular character of function θ. On the other hand, R_k is the "actual infinity" mentioned in the above. On fig. 3 is shown the correspondence between the two sets of coordinates: $\eta=0$ is the free boundary, $\eta=1$ — the numerical infinity, $\xi=0,\ 1$ — the two parts of the axis of symmetry.

The partial derivatives with respect to the new coordinates of the original set are expressed as follows:

(2.4) $$\left|\begin{array}{ll} r_\xi=(1-\eta)R_\xi & r_\eta=R_k-R \\ \theta_\xi=(1-\eta)G_\xi+\pi\eta & \theta_\eta=\pi\xi-G. \end{array}\right.$$

The scale factors (metric coefficients) read:

(2.5) $$\left|\begin{array}{l} H_\xi=\sqrt{r_\xi^2+r^2\theta_\xi^2}=\sqrt{(1-\eta)^2R_\xi^2+R^2[(1-\eta)G_\xi+\pi\eta]^2} \\ H_\eta=\sqrt{r_\eta^2+r^2\theta_\eta^2}=\sqrt{(R_k-R)^2+R^2(\pi\xi-G)^2}. \end{array}\right.$$

At the boundary, the last expressions reduce to:

(2.6) $$H_\xi\big|_{\eta=0}=\sqrt{R_\xi^2+R^2G_\xi^2}, \quad H_\eta\big|_{\eta=0}=\sqrt{(R_k-R)^2+R^2(\pi\xi-G)^2}.$$

Description of the new coordinate system is completed by the formulae for differentiation

(2.7) $$\frac{\partial}{\partial r}=J^{-1}\left(\theta_\eta\frac{\partial}{\partial\xi}-\theta_\xi\frac{\partial}{\partial\eta}\right) \quad \frac{\partial}{\partial\theta}=J^{-1}\left(r_\xi\frac{\partial}{\partial\eta}-r_\eta\frac{\partial}{\partial\xi}\right),$$

where

(2.8) $$J=r_\xi\theta_\eta-r_\eta\theta_\xi$$

is the Jacobian of the transformation calculated on the base of (2.4).

3. Reformulation of the boundary conditions

Having the formulae from previous section one can easily transform the Laplace equation (1.1) into new coordinates. This is a trivial though tedious task which is out of the scope of the present paper. For our purposes is enough to be certain that it can be done. Let us denote:

(3.1) $$\tilde{\varphi}(\xi,\ \eta,\ t)=\varphi(r(\xi,\ \eta,\ t),\ \theta(\xi,\ \eta,\ t),\ t).$$

Then making use of boundary condition (1.2) and (2.7) we obtain the following relation

$$\frac{\tilde{\varphi}_\eta(1-\eta)\,R_\xi(0,\ t)-[R_k-R\ (0,\ t)]\ \tilde{\varphi}_\xi}{(1-\eta)\,R_\xi[\pi\xi-G]-[R_k-R(0,\ t)]\ [(1-\eta)G_\xi(0,\ t)+\pi\eta]}=0.$$

As it will be seen in what follows the problem consists of the second derivatives of functions $R(\xi,\ t)$ and $G(\xi,\ t)$ and therefore requires boundary conditions for these functions at the axis of symmetry. For function G they are already specified in (2.3). For R we require:

(3.2) $$R_\xi(0,\ t)=0 \quad \text{at} \quad \xi=0{,}1,$$

and the above equality adopts then the more simple form:

(3.3) $$\tilde{\varphi}_\xi=0 \quad \text{at} \quad \xi=0{,}1.$$

Boundary condition at infinity remains unchanged and only the independent variables r, θ are replaced by ξ, η, namely

(3.4) $$\tilde{\varphi}=U_\infty R_k \cos\theta(\xi,\ \eta,\ t) \quad \text{at} \quad \eta=1.$$

In terms of new independent variables the shape function of the boundary which corresponds to the moving surface is simply $\tilde{\varphi}(\xi,\ \eta,\ t)=\eta$ since this line is represented by $\eta=0$. Then we have:

(3.5) $$0=\tilde{\Phi}(\xi,\ 0,\ t)\equiv\Phi(r(\xi,\ 0,\ t),\ \theta(\xi,\ 0,\ t),\ t)=0$$

which is in fact automatically satisfied by the very definition of functions R and G as parametric representation of (1.4). Taking the partial derivative with respect to time of (3.5) we get:

$$0=\frac{\partial\tilde{\Phi}}{\partial t}=\frac{\partial\Phi}{\partial t}+r_t\frac{\partial\Phi}{\partial r}+\theta_t\frac{\partial\Phi}{\partial\theta}, \quad \text{at} \quad \eta=0.$$

By means of the last equality aud kinematic condition (1.5) making use of (2.7) we get:

(3.6) $$\frac{1}{J}(\theta_\xi r_t-r_\xi\theta_t)+\frac{1}{J^2R^2}H_\xi^2\frac{\partial\tilde{\varphi}}{\partial\eta}$$
$$-\frac{1}{J^2R^2}(r_\eta r_\xi+r^2\theta_\eta\theta_\xi)\frac{\partial\tilde{\varphi}}{\partial\xi}=0 \quad \text{at} \quad \eta=0.$$

Turning to the dynamic condition we initially derive the following auxilliary relation

$$\frac{\partial\varphi}{\partial t}\equiv\frac{\partial\tilde{\varphi}}{\partial t}+\frac{1}{J}(r_\eta\theta_t-\theta_\eta r_t)\frac{\partial\tilde{\varphi}}{\partial\xi}+\frac{1}{J}(\theta_\xi r_t-r_\xi\theta_t)\frac{\partial\tilde{\varphi}}{\partial\eta}=0$$

and introducing here (3.6) we obtain:

(3.7) $$\frac{\partial\varphi}{\partial t}=\frac{\partial\tilde{\varphi}}{\partial t}-H_\xi^2J^{-2}R^{-2}\varphi_\eta^2+J^{-1}(r_\eta\theta_t-\theta_\eta r_t)\,\tilde{\varphi}_\xi$$
$$+J^{-2}(\theta_\xi\theta_\eta+R^{-2}r_\xi r_\eta)\,\varphi_\xi\,\tilde{\varphi}_\eta \quad \text{at} \quad \eta=0.$$

On the other hand,

$$(3.8)\qquad (\varphi_r^2+r^{-2}\varphi_\theta^2)_{\eta=0}=J^{-2}R^{-2}\{H_\eta^2\varphi_\xi^2+H_\xi^2\varphi_\eta^2$$

$$-2(R^2\theta_\xi\theta_\eta+r_\xi r_\eta)\,\tilde{\varphi}_\xi\,\tilde{\varphi}_\eta\}_{\eta=0}.$$

Substituting all this in (1.6) we have:

$$(3.9)\qquad \frac{\partial\varphi}{\partial t}+H_\xi^2 J^{-2}R^{-2}\tilde{\varphi}_\xi^2-J^{-2}R^{-2}(\theta_\xi\theta_\eta R^2+r_\xi r_\eta)\,\tilde{\varphi}_\xi\,\tilde{\varphi}_\eta$$

$$+J^{-1}(r_\eta\theta_t-\theta_\eta r_t)\tilde{\varphi}_\xi=-gR\cos\theta(\xi,\ \eta,\ t)+F(t)$$

$$-\sigma\frac{R(R_{\xi\xi}G_\xi-R_\xi G_{\xi\xi})}{(R_\xi^2+R^2G_\xi^2)^{3/2}}+\sigma\frac{2R_\xi^2G_\xi+R^2G_\xi^3}{(R_\xi^2+R^2G_\xi^2)^{3/2}}$$

$$+\sigma\frac{|R_\xi\cos G-RG_\xi\sin G|}{R\sin G\,(R_\xi^2+R^2G_\xi^2)^{1/2}},\quad \eta=0,$$

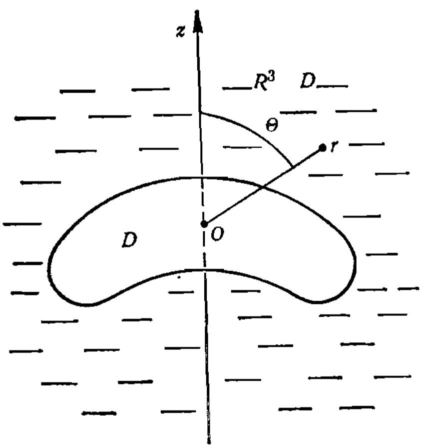
where the curvature is already expressed in terms of functions R and G.

4. Coupling the system on the surface

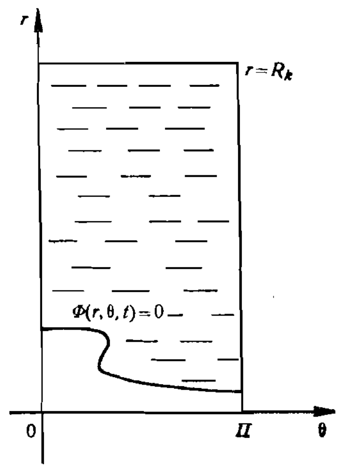
It is clearly seen that the conditions (3.6) and (3.9) are not enough to provide a boundary condition for potential function $\tilde{\varphi}$ and to define the two functions R and G. It is in our disposal to impose one more condition on these two functions in order to get a unique solution for them. For instance, the equation

$$(4.1)\qquad G(\xi,\ t)=\pi\xi$$

reduce the method of transfinite interpolation to the method of scaled variable mentioned in the above. As has been argued in introduction the latter is sub-



Фиг. 1



Фиг. 2

ject to the limitation of single valued shape function. In the case when (1.4) holds this means that R has to be single-valued function of ξ, i. e. the region has to be starwise. For example, region depicted on Fig. 1 and Fig. 2 is not starwise and can not be treated by means of method of scaled variable.

The best way to couple the system at our opinion is to impose a condition on the scale factor H_ξ at the boundary $\eta=0$. The latter offers the golden opportunity to govern the mesh size. It can be done, e. g. as follows:

$$H_\xi|_{\eta=0}=Q^{-1}(\xi, t) \quad \text{or} \quad Q(\xi, t)H_\xi|_{\eta=0}=1, \tag{4.2}$$

where Q^{-1} is the function which is supposed to govern the mesh size (the grid spacing) at the boundary. When $Q(\xi, t)$ is large the scale factor H_ξ becomes small and vice versa. Function $Q(\xi, t)$ can be set to be proportional to curvature, to local gradient of the flow, etc. Taking $Q(\xi, t)=const$ one obtains a mesh which is uniformly spaced along the surface.

It is more convenient to have (4.2) rewritten in the form

$$\frac{dQH_\xi}{d\xi}=0 \quad \text{at} \quad \eta=0, \tag{4.3}$$

and introducing here the first of formulae (2.6) we obtain

$$R_\xi R_{\xi\xi}+RR_\xi G_\xi^2+R^2G_\xi G_{\xi\xi}=-\frac{Q_\xi}{Q}(R_\xi^2+R^2G_\xi^2). \tag{4.4}$$

To this equation have to be added (3.6) and (3.9) rewritten in appropriate form

$$(G_\xi R_t-R_\xi G_t)+J^{-1}R^{-2}(R_\xi^2+R^2G_\xi^2)\frac{\partial\tilde\varphi}{\partial\xi}\Big|_{\eta=0} \tag{4.5}$$

$$-J^{-1}R^{-2}[(\pi\xi-G)G_\xi R^2+(R_k-R)R_\xi]\frac{\partial\tilde\varphi}{\partial\eta}\Big|_{\eta=0}=0.$$

$$\frac{\partial\tilde\varphi}{\partial t}+2J^{-2}R^{-2}\Big\{[(R_k-R)^2+R^2(\pi\xi-G)^2]\left(\frac{\partial\tilde\varphi}{\partial\xi}\right)^2_{\eta=0} \tag{4.6}$$

$$-(R_\xi^2+R^2G_\xi^2)\left(\frac{\partial\tilde\varphi}{\partial\eta}\right)^2_{\eta=0}\Big\}+gR\cos G$$

$$+J^{-1}[(R_k-R)G_t-(\pi\xi-G)R_t]\frac{\partial\tilde\varphi}{\partial\xi}=F(t)$$

$$-\sigma\frac{R(R_{\xi\xi}G_\xi-R_\xi G_{\xi\xi})}{(R_\xi^2+R^2G_\xi^2)^{3/2}}+\sigma\frac{2R_\xi^2G_\xi+R^2G_\xi^2}{(R_\xi^2+R^2G_\xi^2)^{3/2}}$$

$$+\sigma\frac{|R_\xi\cos G-RG_\xi\sin G|}{R\sin G\,(R_\xi^2+R^2G_\xi^2)^{1/2}},$$

where

$$J=(\pi\xi-G)R_\xi-(R_k-G)G_\xi. \tag{4.7}$$

Thus (4.4)—(4.7) form a coupled system for estimating functions R, G and for providing the boundary condition for function $\tilde\varphi$. This is a mixed initial-boundary value problem. The boundary conditions has been discussed in the above. The initial condition has to be of the form

$$R(\xi, 0)=R_0(\xi), \quad G(\xi, 0)=G_0(\xi). \tag{4.8}$$

For completeness we cite here the boundary conditions:

$$R_\xi=0, \quad G=0 \quad \text{at} \quad \xi=0 \tag{4.9}$$

$$R_\xi=0, \quad G=\pi \quad \text{at} \quad \xi=1. \tag{4.10}$$

5. Correct posing the initial value problem

The governing system of equations (4.4)—(4.7) for mesh evolution is complicated one as the equations are not resolved with respect to time derivatives of different functions. The problem here is that one can resolve the system in more than one way and to deduce different systems with different properties. Part of those systems can prove incorrect as an initial value problem. The main objective of present paper is to show the way of correct resolving the original system.

In order to display the main idea without going into immaterial details we take the original set of equations into the following form

$$G_\xi R_t - R_\xi G_t = A(\xi,\ t) \tag{5.1}$$

$$R_\xi R_{\xi\xi} + R^2 G_\xi G_{\xi\xi} + R R_\xi G_\xi^2 = B(\xi,\ t) \tag{5.2}$$

$$\tilde{\varphi}_t + J^{-1}[(R_k - R)G_t - (\pi\xi - R)R_t]\,\tilde{\varphi}_\xi|_{\eta=0} = C(\xi,\ t) \tag{5.3}$$

$$-\sigma R(R_{\xi\xi}G_\xi - R_\xi G_{\xi\xi})(R_\xi^2 + R^2 G_\xi^2)^{-3/2}.$$

At the beginning, it is more instructive to show the way to ob tain ncorreict nitial value problem. All the more that this way seems to be the most inatural one. It consists in deriving equations which contain simultaneously R_t and $R_{\xi\xi}$ or G_t and $G_{\xi\xi}$, respectively. Such equations are obtained through multiplying (5.1) by $J^{-1}(R_k - R)\tilde{\varphi}_\xi|_{\eta=0}$ and adding it to (5.3) which is priory multiplied by R_ξ. Then terms proportional to G_t are canceled and one has

$$R_\xi \tilde{\varphi}_t + J^{-1}[G_\xi(R_k - R) - (\pi\xi - R)R_\xi]\tilde{\varphi}_\xi R_t = R_\xi C(\xi,\ t) \tag{5.4}$$

$$+ J^{-1}\tilde{\varphi}_\xi(R_k - R)A(\xi,\ t) - \sigma R_\xi R(R_{\xi\xi}G_\xi - R_\xi G_{\xi\xi})\ (R_\xi^2 + R^2 G_\xi^2)^{-3/2}.$$

The last equation is multiplied by G_ξ and from the result is substracted (5.2) priory multiplied by $\sigma R_\xi^2 R^{-1}(R_\xi^2 + R^2 G_\xi^2)^{-3/2}$. Then the term proportional to $G_{\xi\xi}$ is canceled. Being also reminded of (4.7) one obtains:

$$R_\xi G_\xi \tilde{\varphi}_t - G_\xi \tilde{\varphi} R_t = R_\xi G_\xi C(\xi,\ t) + J^{-1}\tilde{\varphi}_\xi G_\xi(R_k - R)A(\xi,\ t)$$

$$+ \sigma[R R_\xi G_\xi^2 - B(\xi,\ t)]R_\xi^2 R^{-1}(R_\xi^2 + R^2 G_\xi^2)^{-3/2} - \sigma R_{\xi\xi}(R_\xi^2 + R^2 G_\xi^2)^{-3/2} R^{-1}.$$

Denoting the insignificant terms by $D(\xi,\ t)$ we have

$$G_\xi \tilde{\varphi}_\xi R_t = \sigma R_\xi R^{-1}(R_\xi^2 + R^2 G_\xi^2)^{-1/2} R_{\xi\xi} + D(\xi,\ t). \tag{5.5}$$

In a similar way is obtained that

$$-R_\xi \tilde{\varphi}_\xi G_t = \sigma G_\xi R^2 (R_\xi^2 + R^2 G_\xi^2)^{-1/2} G_{\xi\xi} + E(\xi,\ t), \tag{5.6}$$

where by $E(\xi,\ t)$ are denoted the insignificant for the correctness terms.

It is obvious now that according to the sign of the product $G_\xi R_\xi \tilde{\varphi}_\xi$ either (5.5) or (5.6) becomes an anti-parabolic equation. It is well known that the latter is incorrect as an initial value problem, so that the system (5.5), (5.7) is always incorrect as an initial value problem. This fact means that either the original system (5.1)—(5.3) is incorrect or the incorrectness is introduced by the manipulations. As so it becomes very important to find a transformation, if it exists of course, which yields to a correct system. The main achievement of the present work is that such a correct transformation is found.

Now, we shall transform the system (5.1)—(5.3) in a way to get equations resolved with respect to pairs of derivatives R_t, $G_{\xi\xi}$ and G_t, $R_{\xi\xi}$. For this purpose (5.2) is multiplied by $\sigma R G_\xi (R_\xi^2 + R^2 G_\xi^2)^{-3/2}$ and added to (5.4). As a result is obtained:

(5.7) $$-\tilde{\varphi}_\xi R_t + R_\xi \tilde{\varphi}_t = R_\xi C(\xi, t) + J^{-1}\tilde{\varphi}_\xi (R_k - R) A(\xi, t) + [R R_\xi G_\xi^2 - B(\xi, t)]\sigma R G_\xi (R_\xi^2 + R^2 G_\xi^2)^{3/2} + \sigma R (R_\xi^2 + R^2 G_\xi^2)^{-1/2} G_{\xi\xi}.$$

Respectively, multiplying (5.2) by $\sigma R R_\xi (R_\xi^2 + R^2 G_\xi^2)^{-3/2}$ and substracting it from the appropriately manipulated original set, namely from

(5.8) $$G_\xi \tilde{\varphi}_t + J^{-1}[G_\xi (R_k - R) - (\pi\xi - R) R_\xi]\tilde{\varphi}_\xi G_t = G_\xi C(\xi, t) + \frac{\pi^\xi - R}{J}\tilde{\varphi}_\xi A(\xi, t) - \frac{\sigma R G_\xi}{(R_\xi^2 + R^2 G_\xi^2)^{3/2}} (R_{\xi\xi} G_\xi - R_\xi G_{\xi\xi})$$

we obtain

(5.9) $$-\tilde{\varphi}_\xi G_t + G_\xi \tilde{\varphi}_t = G_\xi C(\xi, t) + J^{-1}\tilde{\varphi}_\xi (\pi\xi - R) A(\xi, t) + [R R_\xi G_\xi^2 - B(\xi, t)]\sigma R_\xi R^{-1} (R_\xi^2 + R^2 G_\xi^2)^{-3/2} - \sigma (R_\xi^2 + R^2 G_\xi^2)^{-1/2} R^{-1} R_{\xi\xi}.$$

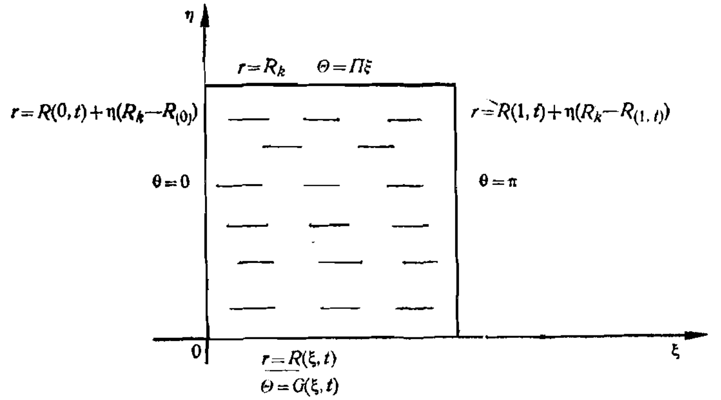


Fig. 3

Once again renaming the insignificant terms we can rewrite the system (5.7), (5.9) in more concise form:

(5.10) $$-\tilde{\varphi}_\xi R_t = \sigma R (R_\xi^2 + R^2 G_\xi^2)^{-1/2} G_{\xi\xi} + P(\xi, t),$$

(5.11) $$-\tilde{\varphi}_\xi G_t = -\sigma R^{-1} (R_\xi^2 + R^2 G_\xi^2)^{-1/2} R_{\xi\xi} + Q(\xi, t).$$

One of the functions here can be excluded (at least its higher derivatives) and to obtain

$$\tilde{\varphi}_\xi^2 R_{tt}+\sigma^2(R_\xi^2+R^2G_\xi^2)R_{\xi\xi\xi\xi}=T(R,\ G,\ \tilde{\varphi}), \tag{5.12}$$

where T is certain complicated function of the lower-order derivatives of functions R and G and derivatives of potential function φ. Since the both coefficients in the left-hand side of (5.12) are positive it is correct as an initial value problem because it is the well known equation of vibration of elastic rods. Initial conditions for this equation are obtained from (4.8) making use of (5.10). The corrolary of correctness of (5.12) is that the original system (5.10) and (5.11) is correct and hence the system (5.1)—(5.3) is correct as initial value problem.

In the end it can be said that the numerical implementation of the problem is most convenient if system (5.10), (5.11) is discretized and solved simultaneously on the new time stage by means of vector Gaussian elimination with pivoting similar to that proposed in [10].

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Introduction

In recent years the fluid flows with free or moving boundaries enjoy a considerable interest both from theoretical and experimental point of view. This is due to their importance in a number of applications, e. g. motions of drops and bubbles through liquids, cavitation, waves on fluid surfaces, etc. At the same time, on the way of theoretical modelling of free-surface flows lie formidable obstacles founded in the strong non-linearity of the respective boundary value problems. That is why the only way to obtain theoretical predictions is the numerical one, except for certain limiting cases for which asymptotic analytical methods can be developed.

The most natural way to construct a numerical algorithm is to introduce new independent variables in a way to get the region of flow fixed and stationary in terms of the new coordinates. Moreover, the moving boundaries become coordinate lines. This approach is so attractive that in last two decades an extensive literature has been published concerning the construction of adaptive meshes (see for detailed survey [1]). Most of these papers, however, are concerned with constructing so-called body-fitted coordinate meshes for bodies with intricated but still fixed shape.

When the boundary of the flow is not priory known, but rather is governed by an equation the complexity of the problem increases significantly since the correctness of such an equation may depend on the way in which the grid is constructed. That is why the advanced grid generation techniques are not yet commonly used in numerical treatment of flows with moving boundaries.

The simplest and most obvious way to obtain boundary-fitted coordinate system is to scale one of the coordinates by the shape function of the boundary. In this case one arrives to well posed coordinate transformation only if the shape function is a single-valued function of the respective coordinate. Regardless to its limitations the method of scaled variable has been successfully applied in [2], [3] to the problem of bubble motion in ideal liquid and in [4] — in viscous liquids. This method proves efficient also for treating the cavitating flows (see [5]). It turned out to be very helpfull in investigation of the global correctness of viscous flows with free boundaries [6].