

AN INVISCID MODEL OF FLOW SEPARATION AROUND BLUNT BODIES

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Despite numerous publications on theoretical modelling of the separated flows a self-contained adequate theory is still undeveloped, because of the formidable difficulties founded in the very nature of the problem. Difficulties arise from the intricate interaction of the effects of rigid and free flow boundaries that conjoin at priory unknown points and rear end of a cavity or stagnation zone. In fact, the attitudes toward the treatment of these two principal points are which make the difference between the different theories (see [1,2] for a comprehensive view of this subject).

The peculiar thing about the semi-empirical approaches to detachment or rear-end points of a cavity is that certain (sometime quite arbitrary) conditions are summoned to define these points or the behaviour of the free surface in their vicinities. In order to overcome these shortcomings in [3], it is proposed to consider the ideal flow as not necessarily potential but rather to admit ideal flows that are potential in some regions and stagnant in others. Such flows are very important since they turn out to be the limiting form of viscous flows with vanishing viscosity [4,5], i. e. they are the 'outer' solutions of asymptotic expansions to which the 'inner' (boundary-layer) solutions must match.

The present paper is a further development of the numerical technique for implementing of the above model of separation.

1. Posing the Problem. Consider the two-dimensional steady inviscid flow around infinite cylinder of arbitrary star-like cross-section in terms of polar coordinates. Let also the flow be symmetric with respect to a plane containing the axis of cylinder. As far as the two-dimensional streaming is concerned, we can consider stream function ψ that is governed by the Laplace equation $\Delta\psi=0$.

Since the cylindrical coordinate system becomes deficient when the stagnation zone elongates, here we choose the parabolic coordinate system which in our opinion is most suited for treating infinite stagnation zones. In its turn, it is obvious that the parabolic coordinates are inconvenient for description of bodies with blunt rear ends. For this reason, we use them only after the stagnation zone is already developed.

Hereafter, we shall compile the necessary formulae for both the coordinate systems in a parallel manner denoting them by a or b , respectively. The Laplace equation for the stream function reads

$$(1a) \quad \frac{\partial^2\psi}{\partial r^2} + \frac{1}{r} \frac{\partial\psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2\psi}{\partial\theta^2} = 0, \quad \text{and} \quad (1b) \quad \frac{1}{\sigma^2 + \tau^2} \left(\frac{\partial^2\psi}{\partial\sigma^2} + \frac{\partial^2\psi}{\partial\tau^2} \right) = 0.$$

These equations are coupled by the respective boundary conditions. At infinity a uniform stream of velocity is present

$$(2a) \quad \psi \sim r U_\infty \sin \theta, \quad r \rightarrow \infty \quad \text{and} \quad (2b) \quad \psi \sim \sigma \tau U_\infty, \quad \sigma \rightarrow \infty, \quad \tau \rightarrow \pm \infty$$

On both portions Γ_1 and Γ_2 of the boundary (Γ_1 representing the surface of the body and Γ_1 -- the boundary of separated zone) the normal flux must be equal to zero which simply gives

$$(3a) \quad \psi = 0 \quad \text{for } r = R(\theta), \quad \theta \in \Gamma \quad \text{and} \quad (3b) \quad \psi = 0 \quad \text{for } \sigma = S(\tau), \quad \sigma \in \Gamma.$$

Only on the boundary Γ_2 of the separated zone in addition to (3) the Bernoulli integral holds, namely

$$(4a) \quad \frac{1}{2} \left[\left(\frac{\partial \psi}{\partial r} \right)^2 + \left(\frac{\partial \psi}{\partial \theta} \right)^2 \right] + \frac{p_c}{\rho} = U_\infty^2 + \frac{p_\infty}{\rho}, \quad r = R(\theta), \quad \theta \in \Gamma_2;$$

$$(4b) \quad \frac{1}{2(\sigma^2 + \tau^2)} \left[\left(\frac{\partial \psi}{\partial \sigma} \right)^2 + \left(\frac{\partial \psi}{\partial \tau} \right)^2 \right] + \frac{p_c}{\rho} = U_\infty^2 + \frac{p_\infty}{\rho}, \quad \sigma = S(\tau), \quad \tau \in \Gamma_2.$$

Here p_c is the magnitude of pressure in cavity (stagnation zone), p_∞ — pressure at infinity, U_∞ — velocity at infinity. The additional boundary condition (4) does not render the problem incorrect, because the shape of the portion Γ_2 of the boundary is unknown.

2. Coordinate Transformation. Though correct, the above boundary value problem is highly inconvenient for numerical treatment due to the fact that the shape of the separated zone is to be implicitly evaluated from the additional boundary condition (4). The most efficient way to solve it is, perhaps, to make use of a coordinate transformation rendering the original domain to a region of known boundaries and thus to derive an explicit equation for the shape function. We introduce the following independent coordinates

$$(5a) \quad \eta = r R^{-1}(\theta); \quad (5b) \quad \eta = \sigma - S(\tau)$$

instead of the polar radius r and parabolic coordinate σ , respectively. In terms of the new coordinates (η, θ) or (η, τ) the Laplace equation adopts the following form

$$(6) \quad a \frac{\partial^2 \psi}{\partial \eta^2} - 2b \frac{\partial^2 \psi}{\partial \eta \partial \xi} + \frac{\partial^2 \psi}{\partial \xi^2} + d \frac{\partial \psi}{\partial \eta} = 0,$$

where

$$(6a) \quad \begin{aligned} \xi &= \theta \\ a &= \eta^2 (1 + (R'/R)^2) \\ b &= \eta R'/R \\ d &= \eta (1 - R''/R + 2(R'/R)^2) \end{aligned} \quad (6b) \quad \begin{aligned} \xi &= \tau \\ a &= 1 + S'^2 \\ b &= S' \\ d &= -S'' \end{aligned}$$

In terms of new coordinates the condition at infinity reads

$$(7a) \quad \begin{aligned} \psi &\sim \eta U_\infty R(\theta) \sin \theta; \\ \eta &\rightarrow \infty \end{aligned} \quad (7b) \quad \begin{aligned} \psi &\sim [\eta + S(\tau)] \tau U_\infty. \\ \eta &\rightarrow \infty \end{aligned}$$

The kinematic boundary condition remains unchanged

$$(8a) \quad \psi = 0 \quad \text{for } \eta = 1; \quad (8b) \quad \psi = 0 \quad \text{for } \eta = 0.$$

Thus we have a correct boundary value problem for evaluating the function ψ if functions $R(\theta)$ or $S(\tau)$ are thought as known. In turn, at the portion Γ_2 of the boundary where they are not known they can be calculated from the Bernoulli integral (4) which becomes an explicit relation for the shape function

$$(9a) \quad \frac{1}{R^2} \left(1 + \left(\frac{R'}{R} \right)^2 \left(\frac{\partial \psi}{\partial \eta} \Big|_{\eta=1} \right)^2 \right) = 1 + \kappa, \quad \theta \in \Gamma_2;$$

$$(9b) \quad \frac{1}{S^2 + \tau^2} (1 + S'^2) \left(\frac{\partial \Psi}{\partial \eta} \Big|_{\eta=0} \right)^2 = 1 + \kappa, \quad \tau \in \Gamma_2,$$

where κ is cavitation number.

3. Difference Scheme for Laplace Equation. The Laplace equation becomes somewhat more complex showing oblique derivatives but it proves to be a little worry for the numerical treatment (for details see [3,6,7]).

4. Difference Scheme for Solving the Equation for Stagnation-zone Shape. Equations (9) can be resolved for the derivatives of unknowns R' or S' if the following conditions hold

$$(10a) \quad v(0) = (1 + \kappa) \frac{R^2}{T^2} - 1 > 0 \quad \text{and} \quad (10b) \quad w(\tau) = (1 + \kappa) \frac{S^2 + \tau^2}{T^2} - 1 > 0,$$

where $T = \frac{\partial \Psi}{\partial \eta}$ is the value at the boundary. The last inequalities are true at least in the vicinities of the leading- and rear-stagnation points. It is clear that at the leading-stagnation point a stagnation zone is not to be expected to occur, so we consider only the rear-end point.

Let us now assume that we have the values of set functions ψ_{ij}^a , R_j^a , S_j^a , T_j^a for the global iteration of number α . Then we check whether (10a) (or (10b)) is satisfied and hereon define the region where (9) must be integrated. Let us denote by j^* the last point where (10a) is satisfied and the $j^* + 1$ is the first one where (10a) is not satisfied. For parabolic coordinates the respective points are j^* -th and $(j^* - 1)$ -th. The shape function R of boundary (polar coordinates) on the new iterative stage is calculated from the following difference equation

$$(11a) \quad R_{j+1}^{a+1} - R_j^a = g_{j+1} R_j^{a+1} \sqrt{(1 + \kappa) \left(\frac{R_j^a + R_{j+1}^a}{T_j^a + T_{j+1}^a} \right)^2 - 1}, \quad j = j^* - 1, \dots, 1$$

with initial condition $R_{j^*}^{a+1} = \bar{R}_{j^*}$ (12a),

where \bar{R} is the shape of the body, $g_{j+1} = \xi_{j+1} - \xi_j$, $\xi = \begin{Bmatrix} \theta \\ \tau \end{Bmatrix}$.

While the calculations with polar coordinates bear a preliminary character, the calculations with parabolic coordinates are meant to give the final result and hence some more care is taken in constructing the difference scheme and algorithm. First of all iterations are incorporated in order to tackle the nonlinearity of the equation (9b), i.e. the unknown function $S(\tau)$ is calculated from the following difference equation

$$(11b) \quad S_{j+1}^{a,k+1} - S_j^{a+1} = \pm g_{j+1} \sqrt{(1 + \kappa) \frac{(S_j^{a,k} + S_{j+1}^{a,k})^2 + (\tau_j + \tau_{j+1})^2}{(T_j + T_{j+1})^2} - 1}$$

with the initial condition $S_{j^*}^{a+1} = \bar{S}_{j^*}$ (12b),

where \bar{S} is the shape of the body.

The significance of the difference scheme (11b) is that for given global iteration $\alpha + 1$ at each point τ_j the supposed to be solution for S_{j+1} is iterated until convergence is obtained with respect to index k in (11b), i.e. the nonlinear difference equation is solved. Let us think that this happens for $k = K$. Then settling $S_{j+1}^{a+1} = S_{j+1}^{a,K}$ we obtain the sought solution for the point τ_{j+1} .

An important feature of (11b) is that it is solved with two different signs of the right-hand side. The sign minus gives a solution with decreasing S which we interpret as the Chaplygin-Kolscher type of cavity [1].

The case with the plus sign is more complicated, since the respective ordinary differential equation for $S(\tau)$ (11) becomes unstable and for large τ the calculated fun-

ction $S(\tau)$ increases rapidly. On the other hand, the original equation (9) is nonlinear and does not contain radicals. This means that after the points where the function under the square root adopts a trivial value, one can take the alternate sign of the radical. Thus we use the plus sign only up to the first point in which the function w



Fig. 1

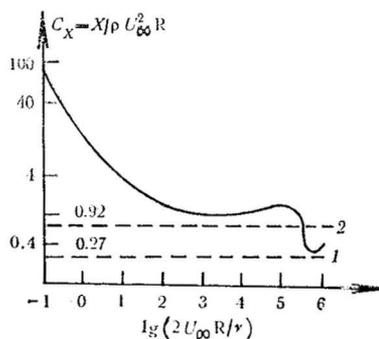


Fig. 2

(τ) (see (10b)) becomes equal to zero. After that (for higher τ) the minus sign is once again employed. In this manner a stable solution is obtained that does not coincide with solution I and we call it solution II and interpret it as the Kirchoff type of cavity [1].

5. Results and Discussion. By means of the above developed algorithm the separated flows around cylinders of arbitrary cross-sections with line of symmetry are computed but we resort to circular cylinder because of the available experimental data to compare with.

In Fig. 1 the stagnation zone obtained for the solution I and II is shown. The shape of solution I can be interpreted as the limiting ideal flow to which is matched the high-Reynolds-number viscous flow, when turbulent boundary layer develops. Respectively, the shape of the solution II is apparently similar to the case of laminar boundary layer (see [8], page 88). This supposition gives excellent prediction for the resistance coefficient of the body (see Fig. 2). Solution II compares with the experimental data ([8], page 25) in the range of Reynolds number in which laminar separation takes place. In its turn, solution I predicts the so-called crisis of resistance, attributed to the transition to turbulence.

The results obtained support the notion that the intrinsic mechanism of the separation phenomenon in an inviscid one and the viscosity plays just the role of a trigger. Another important conclusion is that the essential part of the resistance can be assessed by inviscid model and only for the skin friction viscous flow calculations are needed.

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