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Secondary Streaming for the High-Frequency Viscous Flow in Eccentric Spherical Annuli

1. Introduction

In the recent years the oscillatory viscous flows attracted a considerable attention due to their importance in a number of applications, especially in chemical technology. When an oscillatory streaming interacts with a rigid boundary a secondary steady flow is generated and the latter enhances significantly the heat and mass transfer. On the other hand the investigation of the oscillatory flows offers the opportunity to reveal certain fundamental features of viscous flows which become conspicuous in transient situations with complicated interplay between inertial and viscous forces. The known papers treat different kinds of oscillatory flows and can be divided into two major groups: translatory oscillations and rotational (torsional) oscillations.

Rotational oscillations of single bodies, like disks, spheres, cylinders, etc. in a viscous liquid otherwise at rest has been of special interest to scientists (see e.g. STOKES [1], HELMHOLTZ and PIOTROWSKI [2], BUCHANAN [3], ROSENBLAT [4]). In all these papers only the primary motion of the fluid is considered, viz. it is assumed that the fluid trajectories are circuits whose centres belong to the axis of rotation. The existence of the secondary flow in the plane containing the axis of rotation was shown in papers [5] and [6].

The general approach to oscillatory viscous flows with secondary streamings is essentially developed in papers by STUART [7] and RILEY [8]. The present paper is a descendant from this approach and can be viewed as its application.

The presence of another rigid boundary introduces a lot of difficulties on the way of theoretical treatment which are connected with the geometry of the region occupied by the flow. The most amenable regions to theoretical treatment appear to be those with spherical boundaries at the same time when all the principal features of hydrodynamic interaction are comprised.

Initially, the steady rotations of spheres were investigated asymptotically and numerically for different values of Reynolds number (cf. [9], [10], [11], [12] and [13]). Later on, experiments have been carried out and among the experimental works are renowed the papers by YAWORSKAYA and co-workers (see [14] and literature cited there).

The first papers related to the hydrodynamic interactions between two eccentric spheres treated also stationary rotations. The general approach to this problem based on the use of bi-spherical coordinates was outlined in papers by JEFFERY and co-workers [15], [16] and extended then to the case of a sphere rotating near a plane [17]. The last solution was still generalized by MAJUMDAR [18]. In the above mentioned papers [15]–[18] only the primary flow was considered. MUNSON [19] and MENGUTURK and MUNSON [20] took into account the secondary motion of the fluid obtaining theoretical results and measuring the secondary flow for different values of Reynolds number and eccentricity.

In the recent years there have been obtained interesting theoretical and experimental results for concentric torsional oscillating spheres. The hydrodynamic interaction between two concentric spheres has been considered simultaneously in [21] and [22] for low and high frequencies, respectively. The gap in the theoretical results for the flow considered in [22] has been filled by the numerical solution [23] and now the solution to the problem could be thought of as virtually completed.

Here we discuss the oscillatory flow in eccentric spherical annuli. We focus our attention on the case of high frequency of oscillations which is the more important one to begin with, as in the case of low frequencies one can get some qualitative feeling about patterns of secondary flow even on the base of results for stationary rotations. The method employed here is that of matched singular asymptotic expansions (see VAN DYKE [24]). The differences between the secondary flow streamline patterns in the cases when inner or outer sphere oscillates are discussed and shown graphically.

2. Governing equations

Consider the region between two eccentric spheres of radii a and b ($a < b$) occupied by a viscous incompressible liquid with kinematic coefficient of viscosity ν. It is convenient to make use of bi-spherical coordinates (ξ, η, φ) defined by

$$\varrho' = c\,\frac{\sin\eta}{\operatorname{ch}\xi - \cos\eta}\,, \qquad z' = c\,\frac{\operatorname{sh}\xi}{\operatorname{ch}\xi - \cos\eta} \tag{2.1}$$

were (ϱ', z', φ) are the appropriate polar cylindrical coordinates, c is a positive constant (see for details KORN and KORN [25] and Fig. 1). The inner and the outher spheres correspond to the values $\xi = \alpha_1$ and $\xi = \alpha_2$, respectively.

If we consider an axisymmetrical problem (the axis of oscillations coincides with the line of centres) a stream function can be introduced according to the formulae

$$V'_\xi = \frac{(\operatorname{ch}\xi - \cos\eta)^2}{c^2\sin\eta}\,\frac{\partial\Psi'}{\partial\eta}\,, \qquad V'_\eta = -\frac{(\operatorname{ch}\xi - \cos\eta)^2}{c^2\sin\eta}\,\frac{\partial\Psi'}{\partial\xi}\,. \tag{2.2}$$

Then the Navier-Stokes equations yield the following equations for the dimensionless stream function $\Psi = \Psi'/\varepsilon c^3\omega$ and angular velocity $\Omega = V'_\varphi/\varepsilon c^2\omega$

$$\frac{\partial(D^2\Psi)}{\partial t} + \varepsilon\,\frac{2\Omega(\operatorname{ch}\xi - \cos\eta)^4}{\sin^2\eta}\,\frac{\partial\left(\Omega, \dfrac{\sin\eta}{\operatorname{ch}\xi - \cos\eta}\right)}{\partial(\xi,\eta)} - \varepsilon\,\frac{(\operatorname{ch}\xi - \cos\eta)^3}{\sin\eta}\,\frac{\partial(\Psi, D^2\Psi)}{\partial(\xi,\eta)} + \varepsilon\,\frac{2D^2\Psi(\operatorname{ch}\xi - \cos\eta)^4}{\sin^2\eta}\,\frac{\partial\left(\Psi, \dfrac{\sin\eta}{\operatorname{ch}\xi - \cos\eta}\right)}{\partial(\xi,\eta)} = \frac{1}{M^2}\,D^4\Psi\,, \tag{2.3}$$

$$\frac{\partial\Omega}{\partial t} - \varepsilon\,\frac{(\operatorname{ch}\xi - \cos\eta)^3}{\sin\eta}\,\frac{\partial(\Psi,\Omega)}{\partial(\xi,\eta)} = \frac{1}{M^2}\,D^2\Omega \tag{2.4}$$

where

$$D^2 = \sin\eta(\operatorname{ch}\xi - \cos\eta)\left\{\frac{\partial}{\partial\xi}\left[\frac{(\operatorname{ch}\xi - \cos\eta)}{\sin\eta}\,\frac{\partial}{\partial\xi}\right] + \frac{\partial}{\partial\eta}\left[\frac{(\operatorname{ch}\xi - \cos\eta)}{\sin\eta}\,\frac{\partial}{\partial\eta}\right]\right\}, \qquad D^4 = D^2(D^2)\,.$$

The boundary conditions for Ψ and Ω are:

$$\Psi = \frac{\partial\Psi}{\partial\xi} = 0 \quad \text{at} \quad \xi = \alpha_1\,, \qquad \xi = \alpha_2 \tag{2.5}$$

$$\Omega = \frac{\sin^2\eta}{(\operatorname{ch}\alpha_1 - \cos\eta)^2}\cos t \quad \text{at} \quad \xi = \alpha_1\,, \tag{2.6}$$

$$\Omega = \frac{\sin^2\eta}{(\operatorname{ch}\alpha_2 - \cos\eta)^2}\cos t \quad \text{at} \quad \xi = \alpha_2\,, \tag{2.7}$$

$$\Psi = 0\,, \qquad \Omega = 0 \quad \text{at} \quad \eta = 0, \pi\,. \tag{2.8}$$

The last condition follows from the symmetry relations at the axis of symmetry.

The solution of the problem under consideration depends on two dimensionless parameters, viz. frequency parameter $M = \sqrt{\omega c^2/\nu}$ and amplitude parameter $\varepsilon = \bar{\Omega}/\omega$, where $\bar{\Omega}$ is the amplitude of torsional oscillations and ω their frequency.

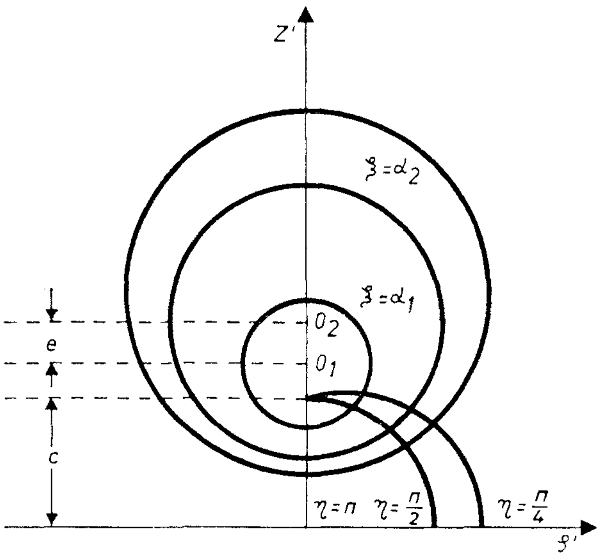

Fig. 1. Abscissa ϱ', ordinate z'

We discuss the case of small amplitude of oscillations and expand the functions Ψ and Ω into power series with respect to the amplitude parameter ε. We restrict our analysis to obtain asymptotic solution of order $O(\varepsilon)$. It is easily shown that $\Psi_0 \equiv 0$. The following system of equations and boundary conditions for Ψ_1 and Ω_0 is derived:

$$\frac{\partial\Omega}{\partial t} = \frac{1}{M^2}\mathcal{D}^2\Omega\,, \tag{2.9}$$

$$\frac{\partial(\mathcal{D}^2\Psi)}{\partial t} - \frac{\operatorname{Re}[\Omega]\,(\operatorname{ch}\xi-\mu)^4}{\sqrt{1-\mu^2}}\,\frac{\partial\left(\operatorname{Re}[\Omega],\ \dfrac{\sqrt{1-\mu^2}}{\operatorname{ch}\xi-\mu}\right)}{\partial(\xi,\mu)} = \frac{1}{M^2}\mathcal{D}^4\Psi, \tag{2.10}$$

$$\Psi = \partial\Psi/\partial\xi = 0 \quad \text{at} \quad \xi = \alpha_1\,, \qquad \xi = \alpha_2 \tag{2.11}$$

$$\Omega = \frac{1-\mu^2}{(\operatorname{ch}\alpha_1-\mu)^2}\,e^{it} \quad \text{at} \quad \xi = \alpha_1\,, \tag{2.12}$$

$$\Omega = \frac{1-\mu^2}{(\operatorname{ch}\alpha_2-\mu)^2}\,e^{it} \quad \text{at} \quad \xi = \alpha_2\,, \tag{2.13}$$

$$\Psi = 0\,, \qquad \Omega = 0 \quad \text{at} \quad \mu = \pm 1\,, \tag{2.14}$$

where

$$\mathcal{D}^2 \equiv (\operatorname{ch}\xi-\mu)\left\{\frac{\partial}{\partial\xi}\left[(\operatorname{ch}\xi-\mu)\frac{\partial}{\partial\xi}\right] + (1-\mu^2)\frac{\partial}{\partial\mu}\left[(\operatorname{ch}\xi-\mu)\frac{\partial}{\partial\mu}\right]\right\},$$

$$\mu = \cos\eta\,, \qquad \mathcal{D}^4 = \mathcal{D}^2(\mathcal{D}^2)\,.$$

In (2.9)–(2.14) it is introduced for convenience the complex-valued functions Ω and e^{it}, and the subscripts for Ψ and Ω are omitted for brevity.

3. Method of matched asymptotic expansions

We attempt an approximate solution to the equations (2.9), (2.10) under the assumption that $M^{-1} \ll 1$. The latter allows us to expand Ψ and Ω into asymptotic series with respect to small parameter M^{-1}. The parameter M^{-1} multiplies the term with the higher-order spatial derivatives which means that the respective series shall be singular one. So, the flow consists of three regions — two boundary layers at spheres walls and a core flow between them. There are three different situations in these three regions and we denote them by $\Psi^{(i)}$, $\Omega^{(i)}$, where $i = 0$ refers to the core flow, $i = 1$ to the boundary layer at the inner sphere ($\xi = \alpha_1$) and $i = 2$ to the outer sphere ($\xi = \alpha_2$) (see Fig. 1).

The scaled coordinates in the boundary layers at the inner and the outer spheres, respectively, are the following

$$\zeta = (\alpha_1 - \xi)\,\frac{M}{\sqrt{2}}\,, \tag{3.1}$$

$$\bar{\zeta} = (\xi - \alpha_2)\,\frac{M}{\sqrt{2}}\,. \tag{3.2}$$

The asymptotic expansions for the functions Ω and Ψ are

$$\Omega^{(i)} = \sum_{n=1}^{\infty} \Omega_n^{(i)}(\xi,\mu,t)\,\alpha_n^{(i)}(M^{-1})\,, \tag{3.3}$$

$$\Psi^{(i)} = \sum_{n=1}^{\infty} \Psi_n^{(i)}(\xi,\mu,t)\,\beta_n^{(i)}(M^{-1})\,, \tag{3.4}$$

where

$$\lim_{M^{-1}\to 0}\frac{\alpha_{n+1}^{(i)}(M^{-1})}{\alpha_n^{(i)}(M^{-1})} = 0\,, \qquad \lim_{M^{-1}\to 0}\frac{\beta_{n+1}^{(i)}(M^{-1})}{\beta_n^{(i)}(M^{-1})} = 0\,.$$

Following the matching procedure (see [24]) for the angular velocity we derive the expressions

$$\Omega_0^{(1)} = \frac{1-\mu^2}{(\operatorname{ch}\alpha_1-\mu)^2}\exp\left(it - \frac{i+1}{\operatorname{ch}\alpha_1-\mu}\,\zeta\right), \qquad \Omega_0^{(0)} = 0\,,$$

$$\Omega_0^{(2)} = \frac{1-\mu^2}{(\operatorname{ch}\alpha_2-\mu)^2}\exp\left(it - \frac{i+1}{\operatorname{ch}\alpha_2-\mu}\,\bar{\zeta}\right),$$

$$\Omega_1^{(1)} = -\frac{(i+1)(1-\mu^2)\operatorname{sh}\alpha_1}{\sqrt{2}\,(\operatorname{ch}\alpha_1-\mu)^4}\,\zeta^2\exp\left(it - \frac{i+1}{\operatorname{ch}\alpha_1-\mu}\,\zeta\right),$$

$$\Omega_1^{(0)} = 0\,, \qquad \Omega_1^{(1)} = \frac{(i+1)(1-\mu^2)\operatorname{sh}\alpha_2}{\sqrt{2}\,(\operatorname{ch}\alpha_2-\mu)^4} \times \bar{\zeta}^2\exp\left(it - \frac{i+1}{\operatorname{ch}\alpha_2-\mu}\,\bar{\zeta}\right).$$

Turning to (2.10) we can notice that the solution for Ψ should be comprised the steady and unsteady part, namely,

$$\Psi = \bar{\Psi}(\xi,\mu) + e^{2it}\,\bar{\bar{\Psi}}(\xi,\mu)\,. \tag{3.5}$$

Indeed (2.10) contains terms proportional to

$$\operatorname{Re}[e^{it}]\operatorname{Re}[e^{it}] = \tfrac{1}{2}\operatorname{Re}(1 + e^{2it})$$

which is the explanation of the form (3.5).

The equation for the function $\bar{\Psi}_0^{(0)}$ (the first-order approximation of the steady streaming in the core flow), which is of interest, is

$$\mathcal{D}^4\bar{\Psi}_0^{(0)} = 0\,. \tag{3.6}$$

After applying the matching procedure for the solution of equation (3.6) we obtain (see [16])

$$\bar{\Psi}_0^{(0)} = (\operatorname{ch}\xi-\mu)^{-3/2}\sum_{n=1}^{\infty}\left[A_n\operatorname{ch}\left(n-\tfrac{1}{2}\right)\xi + B_n\operatorname{sh}\left(n-\tfrac{1}{2}\right)\xi + C_n\operatorname{ch}\left(n+\tfrac{3}{2}\right)\xi + D_n\operatorname{sh}\left(n+\tfrac{3}{2}\right)\xi\right]C_{n+1}^{-1/2}(\mu)\,, \tag{3.7}$$

where $C_{n+1}^{-1/2}(\mu)$ are the Gegenbauer polynomials of order $n+1$ and degree $-1/2$; the coefficients A_n, B_n, C_n and D_n are presented by the following expressions

$$A_n\operatorname{ch}\left(n-\tfrac{1}{2}\right)\alpha_1 + B_n\operatorname{sh}\left(n-\tfrac{1}{2}\right)\alpha_1 + C_n\operatorname{ch}\left(n+\tfrac{3}{2}\right)\alpha_1 + D_n\operatorname{sh}\left(n+\tfrac{3}{2}\right)\alpha_1 = 0\,,$$

$$(2n-1)\left[A_n\operatorname{sh}\left(n-\tfrac{1}{2}\right)\alpha_1 + B_n\operatorname{ch}\left(n-\tfrac{1}{2}\right)\alpha_1\right] + (2n+3)\left[C_n\operatorname{sh}\left(n+\tfrac{3}{2}\right)\alpha_1 + D_n\operatorname{ch}\left(n+\tfrac{3}{2}\right)\alpha_1\right] = a_n(\alpha_1)\,,$$

$$A_n\operatorname{ch}\left(n-\tfrac{1}{2}\right)\alpha_2 + B_n\operatorname{sh}\left(n-\tfrac{1}{2}\right)\alpha_2 + C_n\operatorname{ch}\left(n+\tfrac{3}{2}\right)\alpha_2 + D_n\operatorname{sh}\left(n+\tfrac{3}{2}\right)\alpha_2 = 0\,,$$

$$(2n-1)\left[A_n\operatorname{sh}\left(n-\tfrac{1}{2}\right)\alpha_2 + B_n\operatorname{ch}\left(n-\tfrac{1}{2}\right)\alpha_2\right] + (2n+3)\left[C_n\operatorname{sh}\left(n+\tfrac{3}{2}\right)\alpha_2 + D_n\operatorname{ch}\left(n+\tfrac{3}{2}\right)\alpha_2\right] = a_n(\alpha_2)\,,$$

$$a_n(\alpha_1) = \frac{(2\,e^{-\alpha_1})^{3/2}}{2}\left[n(n+1)\,e^{-\alpha_1 n} + \frac{e^{-2\alpha_1}-1}{3}\,(n-1)\,n\,(n+1)\,e^{-\alpha_1(n-2)}\right],$$

$$a_n(\alpha_2) = \frac{(2\,e^{-\alpha_2})^{3/2}}{2}\left[n(n+1)\,e^{-\alpha_2 n} + \right.$$
$$\left. + \frac{e^{-2\alpha_2}-1}{3}(n-1)\,n(n+1)\,e^{-\alpha_2(n-2)}\right].$$

4. Numerical results and discussion

The numerical implementation of the solution of Section 3 is achieved by calculating on a given grid the values of Ψ on the base of (3.7) with expressions for A_n, B_n, C_n and D_n. Then the obtained two-dimensional array is treated by means of the standard numerical procedure for approximate tracing of the equilines of a function presented in discrete way. In order to have good accuracy for large values of eccentricity e (or which is the same — small c, Fig. 1) we employ a non-uniform mesh in the η-direction according to the rule

$$\left.\begin{aligned}\eta &= 2\,\mathrm{arctg}\left[\frac{\mathrm{sh}\,\alpha_2}{\mathrm{ch}\,\alpha_2+1}\,\mathrm{tg}\,\frac{s.\,\mathrm{sh}\,\alpha_2}{2c}\right],\\ s &= (j-1)\frac{c\pi}{(N-1)\,\mathrm{sh}\,\alpha_2}.\end{aligned}\right\} \tag{4.1}$$

The latter secures that the reference points are uniformly spaced along a meridian of the outer sphere. Parameters of the mesh are $i = 1, \dots, M$ and $j = 1, \dots, N$, for which

$$\xi_i = \alpha_2 + (i-1)\,h_\xi\,, \qquad \eta_j = 0 + (j-1)\,h_\eta\,, \tag{4.2}$$

where

$$h_\xi = (\alpha_1 - \alpha_2)/(M-1)\,, \qquad h_\eta = \pi/(N-1)\,.$$

Computations are run with a couple of different pairs of M, N in order to check the accuracy of the procedure which is responsible for tracing the equi-lines (in this case stream lines). It turned out that $N = 41$ and $M = 51$ is the optimal mesh size, because the employment of more fine meshes does not improve significantly the results and only raises the magnitude of computational time.

The first result to be obtained is for the case of small eccentricity e and fixed radius ratio $\lambda = b/a$. For $c = 100$ (e.g. very small e) our results compare with the computations [22] up to the fourth digid and we have reasonable good comparison within 2% accuracy even for $c = 10$.

Figures 2 and 3 show the dimensionless streamlines for the following three different cases: a) the inner sphere oscillates and the outer one is held at rest; b) the both spheres oscillate; c) the inner sphere is fixed and the outer one oscillates. Fig. 2 corresponds to moderate value of eccentricity e ($c = 1$) and $\lambda = 2{,}5$ and Fig. 3 to large value of eccentricity e ($c = 0{,}2$) and $\lambda = 1{,}5$. Fig. 2, b shows that the negative streamlines coalesce with each other and form a flow known as "cat eye" (see for definition [26]). In the case, which is presented on fig. 3, b, only one positive vortex is observed.

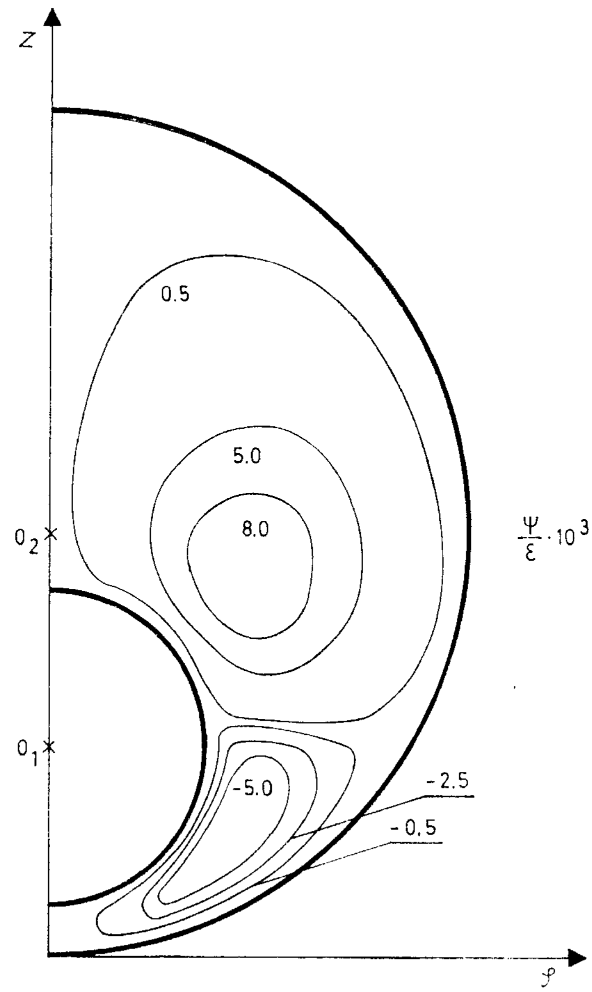

Fig. 2a. Abscissa ϱ, ordinate z; also in the other figures

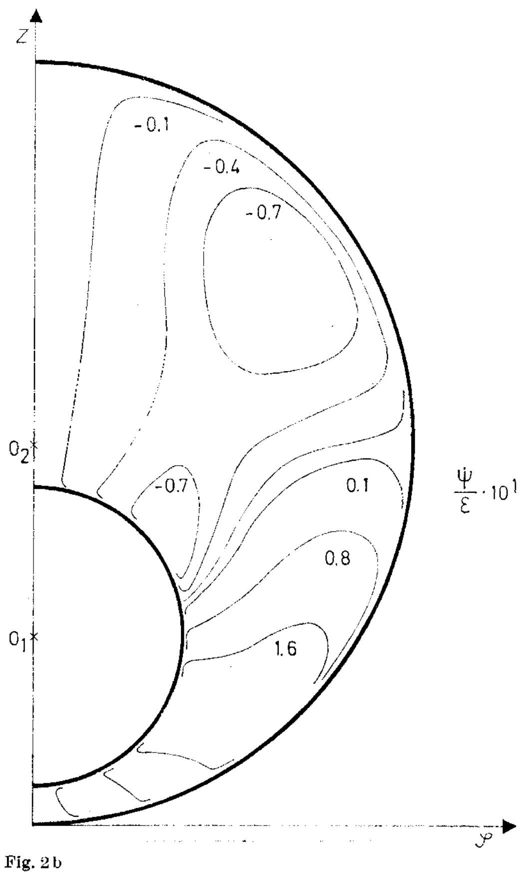

Fig. 2b

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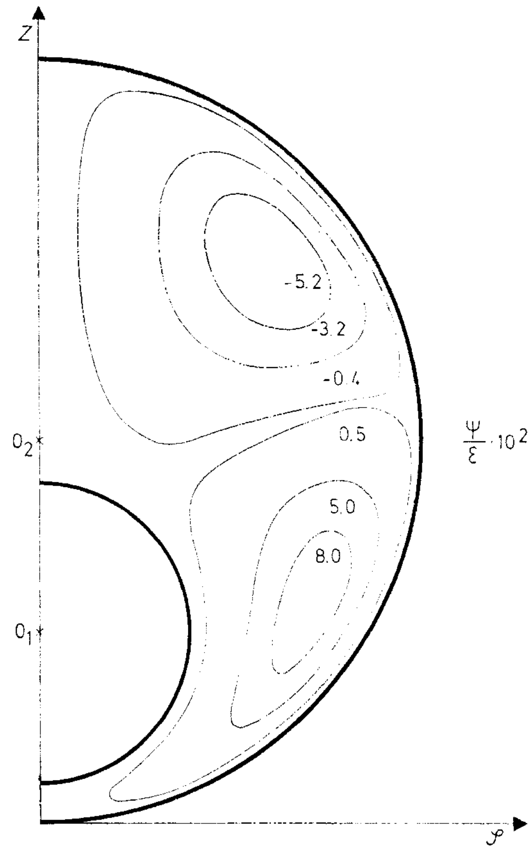

Fig. 2c

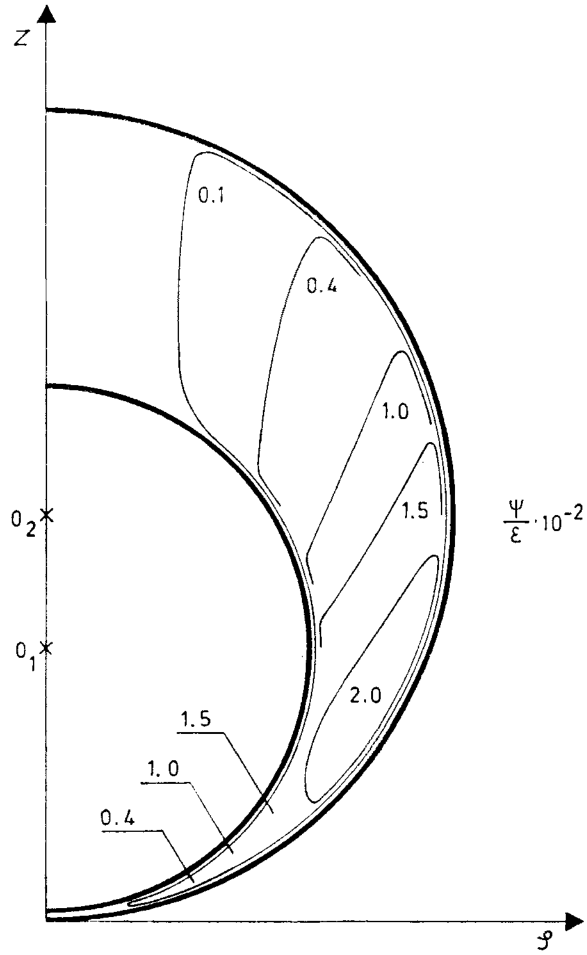

Fig. 3b

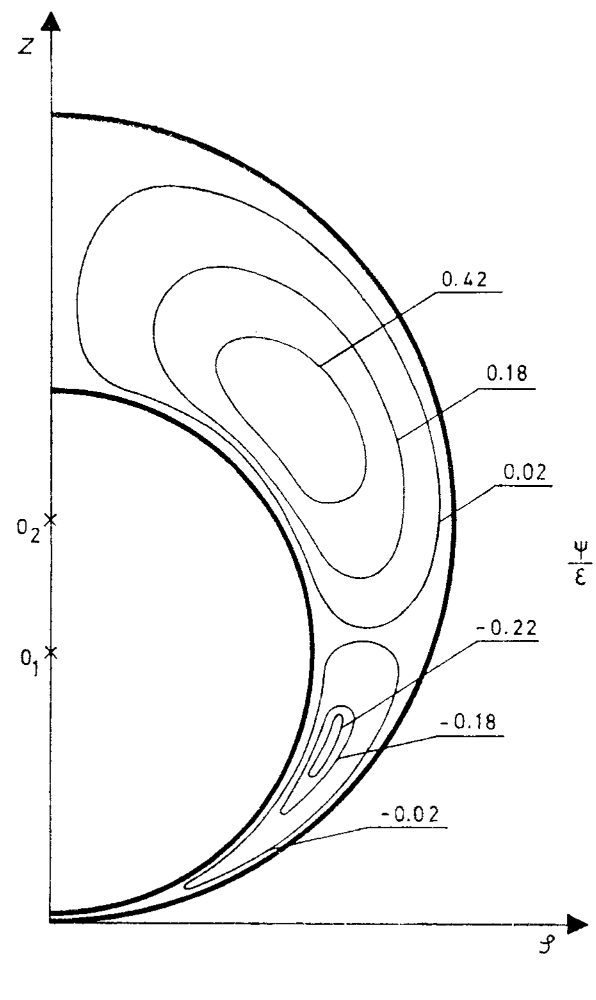

Fig. 3a

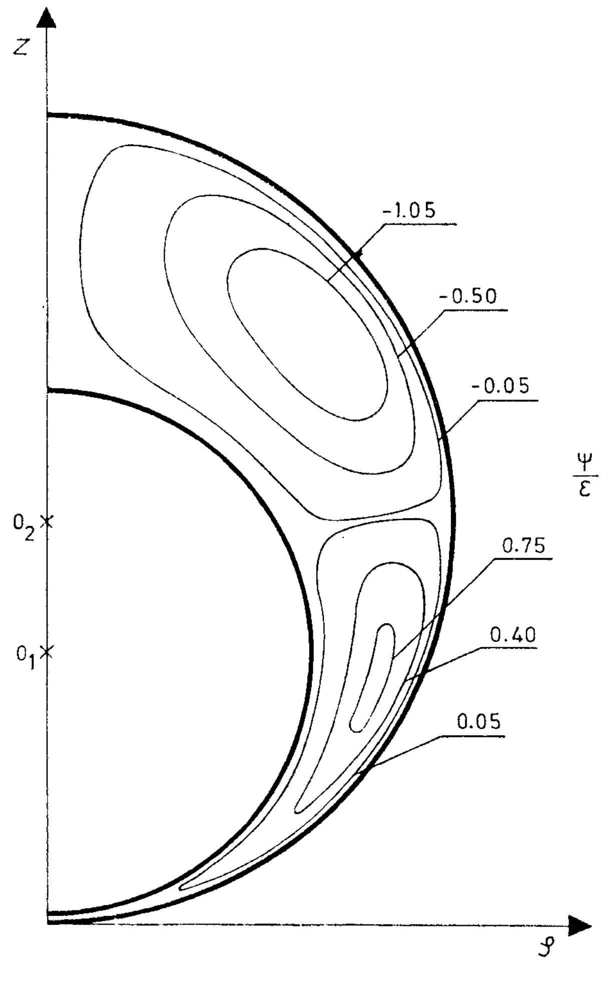

Fig. 3c

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