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ON THE MECHANICS OF LOCALIZED STRUCTURES IN CONTINUOUS MEDIA

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ABSTRACT

From the point of view of soliton paradigm, the notion of discrete versus continuous is revisited. As a featuring example is considered a very thin layer of Hookean elastic medium: a special kind of N–dimensional shell called *gossamer*. It is shown that the linearized equations for the laminar displacements have as corollary the Maxwell equations for appropriately defined quantities called electric and magnetic fields. A higher–order dispersive and nonlinear Boussinesq equation is derived for the flexural deformations of *gossamer*. Due to the conservative properties of Boussinesq equation the flexural solitary waves (*flexons*) are solitons and are identified as particles. The Hamiltonian properties of Boussinesq equation (the metadynamics) define the Hamiltonian (Newtonian) dynamics of the phase objects (*particles–flexons*). Thus a reversed de–Broglie wave–particle dichotomy is introduced. Numerical solutions for the shapes of *flexons* are obtained by means of Method of Variational Imbedding. Localized shear waves of integer–valued topological charge are discussed and identified as *charges*. The membrane tension in the *gossamer* creates between the localized flexural deflections (particles) attractive force (*gravitation*) proportional to r^{1-N}.

1. Introduction

One of the most fascinating concepts of modern physics is the notion of wave–particle dichotomy. In the twenties, de Broglie associated a wave to a particle in order to explain the difraction and interference of particle beams. Another concept appeared after Zabusky and Kruskal[26] discovered in a numerical experiment with the KdV equation that solitary waves and wave crests can behave as particles upon collisions. They called *solitons* these traveling localized structures (particles–waves). In the three decades that followed their work, the solitons received enormous attention and they are hot topic of the nowadays mathematical physics. Yet, the extreme obsession with the mathematical peculiarities of the new object obscured the vision and the really revolutionary aspect of the new concept (the reversion of the de Broglie wave–particle dualism) received little attention or has been often overlooked.

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At the same time in wave mechanics the notion of field became dominant and the quantization of the different fields appeared as the main instrument for investigating the realm of particle physics. Yet little has been done to unify these different aspects of the discrete–versus–continuous dichotomy. In our opinion the notion of soliton (we shall call it in what follows "soliton paradigm") offers a unique opportunity for conceptual unification of the discrete and continuous facets of the physical reality.

In the present work we put forward the idea that all kinds of physical interactions are transmitted by a continuous medium in the sense of classical–mechanics. The objects perceived as discrete (particles, charges, etc) are in fact the localized phase patterns propagating *on* the underlying continuum, the latter playing the role of a "meta" object in the sense that it is beyond (underneath, $\mu\varepsilon\theta\alpha$, etc). The intrinsic properties of the metacontinuum (or the "metadynamics") define the laws of interactions between the localized objects (*solitons* or waves–particles) through the respective Hamiltonian structure which is easily translated to a discrete Hamiltonian for the centers of *solitons*.

Our particular choice for a metacontinuum is the Hookean elastic medium because we have found that for it an equivalent formulation is possible that we call "Maxwell form". Defining an "electric field" as the trace of the stress tensor of the elastic metacontinuum and a "magnetic field" as the **curl** of the velocity field, we show that the well known Maxwell equations can be derived as corollaries from the linearized governing equations of the Hookean elastic medium, provided that the dilational elasticity coefficient is extremely large. Then the shear coefficient defines the speed of shear waves which are naturally called *light*. The advantage of the metacontinuum formulation is that it shows a clear way on how to generalize the Maxwell equations in order to get a model for a field that is *Galilean* invariant. In addition, the enigmatic "field–in–itself" from the Maxwell paradigm is replaced by a banal and down–to–earth mechanical concept – the model of elastic continuous medium.

Justifying thus our choice for the rheology of the medium we proceed further and consider a metacontinuum which is in fact a very thin (generally curved) $N^{\mathcal{D}}$ elastic layer (a shell) in the $(N+1)^{\mathcal{D}}$ geometrical space. Then a geometrically nonlinear theory is developed for extremely thin elastic layers of extremely high stiffness subjected to very large curvatures, moderate deformations and small deflections (we call these layers *gossamers*). As a result a cubic–nonlinear fourth–order dispersive Boussinesq equation appears as the "master" equation for the wave–mechanics of the imaginary Universe under consideration. It is a counterpart to what is known in wave mechanics as Schrödinger equation. Boussinesq equation derived here describes *gossamer* deflections in direction of $N+1$–st dimension. The amplitude of these deflections is the wave function of the wave mechanics considered here. The soliton solutions of the Boussinesq equation are interpreted as the particles. Together with the localized vortex type solutions in the middle surface of the *gossamer* they form the "gross matter". Different properties of the solitons are discussed, including the mutual attraction proportional to r^{1-N}, where N is the dimension of the middle surface of *gossamer*. For a three–dimensional world the attraction is proportional to the inverse square of distance between the centers of objects. Thus the interactions between the

flexural solitons are explained by the curvature of their physical space (in this case it is the *gossamer*).

The solitons of the "master equation" are found numerically by means of a method earlier developed by the author and called Method of Variational Imbedding.

Here we demonstrate the interrelation between the (meta)dynamics of the underlying (meta)continuum and the dynamics of the phase objects (localized waves/solitons) of the same continuum.

2. A Model for a Continuum

If one is to consider the material nonlinearity of the metacontinuum, the Lagrangian description need to be used. We are being guided by the intuitive notion that if the *gossamer* is extremely thin in one of the dimensions, then the material nonlinearity in the middle surface should not matter and we look for a model that is more suited for incorporating the strong geometrical nonlinearity due to flexural deformations of the middle surface. Then we can afford using the Eulerian description which is sufficient to this end while being more tractable and technically simpler.

2.1. Cauchy Equations of a Continuum

In Cauchy form the momentum equations for a continuum read

$$\frac{\partial \mu \mathbf{A}}{\partial t} + \mathbf{A} \cdot \nabla(\mu \mathbf{A}) = -\nabla \varphi + \nabla \cdot \tau \tag{2.1}$$

where $\mathbf{A}$ is the velocity of the continuum; φ is the pressure and μ – density. Let also τ stand for the deviator tensor of internal stresses. The Cauchy momentum equations are coupled with the continuity equation

$$\frac{\partial \mu}{\partial t} + \nabla \cdot (\mu \mathbf{A}) = 0. \tag{2.2}$$

2.2. Hookean Elastic Medium

For elastic body one has the following constitutive relation (the Hooke law)[23]

$$\tau = \eta(\nabla \mathbf{u} + \nabla \mathbf{u}^T) + \lambda(\nabla \cdot \mathbf{u})\mathcal{I}, \tag{2.3}$$

where $\mathbf{u}$ is the displacement vector and η, λ are Lamé elasticity coefficients. For the divergence of the stress tensor τ one obtains

$$\nabla \cdot \tau = \eta \Delta \mathbf{u} + (\lambda + \eta)\nabla(\nabla \cdot \mathbf{u}) \equiv -\eta \nabla \times (\nabla \times \mathbf{u}) + (2\eta + \lambda)\nabla(\nabla \cdot \mathbf{u}). \tag{2.4}$$

For the sake of further convenience we cite the governing equations for an elastic continuum in a form containing both displacements $\mathbf{u}$ and velocities $\mathbf{A}$:

$$\mu_0\left(\frac{\partial \mathbf{A}}{\partial t} + \mathbf{A} \cdot \nabla \mathbf{A}\right) = \eta \Delta \mathbf{u} + (\lambda + \eta)\nabla(\nabla \cdot \mathbf{u}) \ . \tag{2.5}$$

For small velocities on has $|\mathbf{A}\cdot\nabla\mathbf{A}| \ll |\mathbf{A}|$ and the linearization of (2.5) gives,

$$\mu_0\frac{\partial^2\mathbf{u}}{\partial t^2} \approx \mu_0\frac{\partial\mathbf{A}}{\partial t} = \eta\Delta\mathbf{u} + (\lambda+\eta)\nabla(\nabla\cdot\mathbf{u}) \ . \tag{2.6}$$

2.3. *The Shear and Compression Waves: Light and Sound*

Consider the propagation of small disturbances (waves of small amplitude). The full set of physical motions governed by (2.6) includes shear and compression/dilation as well. The former are controlled by the shear Lamé coefficient η, while the latter by the second (dilational) Lamé coefficient λ, and more specifically by the sum $(\lambda+\eta)$. The two kinds of elastic waves separate clearly and the phase velocities of propagation of the respective small disturbances are

$$c \equiv c_l = \left(\frac{\eta}{\mu_0}\right)^{\frac{1}{2}} \qquad c_s = \left(\frac{\eta+\lambda}{\mu_0}\right)^{\frac{1}{2}} \ . \tag{2.7}$$

Here $c = c_l$ is the speed of shear waves (*light*) and c_s – that of compression waves (*sound*).

2.4. *The Case of Very Large Dilational Modulus*

Two extreme limiting cases appear in the elastic continuum: very small or very large dilational coefficient λ. Loosely speaking, the first case describes a strongly compressible continuum while in the second case the continuum is virtually incompressible. For the latter the speed of sound is much greater than that of light which allows introducing the following small parameter

$$\delta = \sqrt{\frac{c_l}{c_s}} \equiv \frac{\eta}{(\eta+\lambda)} \ , \tag{2.8}$$

and recasting eq.(2.5) to the following

$$\delta\left[c^{-2}\left(\frac{\partial\mathbf{A}}{\partial t} + \mathbf{A}\cdot\nabla\mathbf{A}\right) - \Delta\mathbf{u}\right] = \nabla(\nabla\cdot\mathbf{u}) \ . \tag{2.9}$$

We develop the displacement $\mathbf{u}$ in power asymptotic series with respect to δ which we use as an ordering parameter, namely

$$\mathbf{u} = \mathbf{u}_0 + \delta\mathbf{u}_1 + \dots \ . \tag{2.10}$$

Introducing (2.10) into (2.9) and combining the terms with like powers we obtain from the first two approximations

$$\nabla(\nabla\cdot\mathbf{u}_0) \ = \ 0 \ , \tag{2.11}$$

$$c^{-2}\left(\frac{\partial\mathbf{A}_0}{\partial t} + \mathbf{A}_0\cdot\nabla\mathbf{A}_0\right) - \Delta\mathbf{u}_0 \ = \ \nabla(\nabla\cdot\mathbf{u}_1). \tag{2.12}$$

From (2.11) one can deduce the linear approximation of the incompressibility condition for metacontinuum

$$\nabla\cdot\mathbf{u}_0 = \ \text{const}, \qquad \text{or} \qquad \nabla\cdot\mathbf{A}_0 = 0 \ .$$

Denoting formally the term $\nabla\cdot\mathbf{u}_1$ by $(-\varphi)$ one can recast (2.12)

$$c^{-2}\left(\frac{\partial\mathbf{A}_0}{\partial t} + \mathbf{A}_0\cdot\nabla\mathbf{A}_0\right) = -\nabla\varphi + \Delta\mathbf{u}_0 \ , \tag{2.13}$$

which is in fact a dimensionless form of (2.5).

3. Linearized Model of Continuum: Maxwell Form†

3.1. *Maxwell Equations for Incompressible Elastic Medium*

It is interesting to note that the linearized version of the Cauchy form (2.1) has as a corollary the first Maxwell equation (Faraday's law) provided that an "electric field" $\mathbf{E}$ is defined as

$$\mathbf{E} \equiv -\nabla\cdot\tau \ . \tag{3.14}$$

Vector $\mathbf{E}$ has the obvious meaning of a pointwise distributed body force to which the action of the internal stresses is reduced. In terms of $\mathbf{E}$, the linearized system (2.6) yields

$$\mathbf{E} = -\frac{\partial\mu\mathbf{A}}{\partial t} - \nabla\varphi, \tag{3.15}$$

which identifies the well known vector and scalar potentials $\mathbf{A}$ and φ, respectively. Thus from the point of view of the underlying metacontinuum (carrier of the interactions), these potentials are not certain non-physical quantities introduced merely for convenience, but rather they appear to be the most natural variables: velocity and pressure of metacontinuum.

Taking the *curl* of (3.15) one obtains

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \tag{3.16}$$

This equation is nothing else but the first of Maxwell's equation (the Faraday law) provided that a "magnetic induction" $\mathbf{B}$ is defined as

$$\mathbf{B} = \nabla\times(\mu\mathbf{A}). \tag{3.17}$$

†The derivations of the present Sections are not to be confused with McCullagh's model of pseudo-elastic continuum with restoring couples by means of which he tried to explain the apparently unusual shape of Maxwell's equations[18].

Let us consider the above described case of incompressible metacontinuum. In the linear approximation this means that[‡]

$$\nabla \cdot \mathbf{u} = \text{ const} . \qquad (3.18)$$

we obtain for the quantity we called "electric field" $\mathbf{E}$ the simple expression

$$-\mathbf{E} = -\eta \nabla \times (\nabla \times \mathbf{u}). \qquad (3.19)$$

As the magnetic field is equal to magnetic induction scaled by the magnetic permeability μ we define the former as the rotation of the velocity field, namely

$$\mathbf{H} = \nabla \times \mathbf{A} \equiv \nabla \times (\frac{\partial \mathbf{u}}{\partial t}) . \qquad (3.20)$$

Consider the linearized version of the full derivative of $\mathbf{E}$ with respect to time which is simply the partial (local) time derivative. Then (3.19), (3.20) yield

$$\nabla \times \mathbf{H} = \frac{1}{\eta} \frac{\partial \mathbf{E}}{\partial t} , \qquad (3.21)$$

where $\mathbf{H}$ stands for the magnetic field. Eq. (3.21) is nothing else but the so-called "second Maxwell equation" provided that the shear elastic modulus of metacontinuum is interpreted as the inverse of electric permittivity, namely

$$\epsilon_0 = \frac{1}{\eta} \qquad (3.22)$$

The "second Maxwell's equation" was postulated by Maxwell as an improvement over Ampere's law when a term responsible for the so-called displacement current is incorporated in the Biot–Savart form. For the case of a void space, however, when no charges or currents are present the second Maxwell equation lives a life of its own and Ampere's law plays merely heuristic role for its derivation. It is broadly accepted now that the second Maxwell equation is verified by a number of experiments. Here we have shown that it is also a corollary of the elastic rheology of the metacontinuum.

The two main equations of evolution of Maxwell's form have already been derived. The condition div$\mathbf{H} = 0$ is a straightforward corollary from the very definition of magnetic field. Similarly, taking the *div* of (3.19) one immediately obtains that

$$\text{div}\mathbf{E} = 0 , \qquad (3.23)$$

which is the fourth Maxwell equation.

Thus we have shown that Maxwell's equations follow from the governing equations of a Hookean elastic medium provided that the dilational modulus of the latter is much larger than the shear one. Within the linear approximation for the constitutive relation the latter implies that the medium is virtually incompressible.

[‡]When finite deformations and deflections are concerned, the incompressibility condition is more complicated and involves the Jacobian and other invariants of the deformation, or which is the same, of the transformation from material to geometrical variables.

3.2. Role of Convective Nonlinearity: Lorentz Force

Let us consider what kind of effects are to be expected due to the convective nonlinearity of motions in metacontinuum.

The governing equations of the metacontinuum are *Galilean* invariant, while the linearized version (2.6) (and hence, the Maxwell form) have lost this important property. Then arises the question, whether an observer in the $N^{\mathcal{D}}$ space can get a hunch about the type of invariance of the underlying metacontinuum from the manifestation of the physical interactions in $N^{\mathcal{D}}$ space. In order to shed some light on this subject we consider eq.(2.5) in so-called Lamb's form (see,e.g.,[23])

$$\mu_0(\frac{\partial \mathbf{A}}{\partial t} + \frac{1}{2}\nabla|\mathbf{A}|^2 - \mathbf{A} \times \mathbf{rot}\mathbf{A}) + \nabla\varphi = -\mathbf{E} . \qquad (3.24)$$

This form allows us to assess the additional to electrical field forces acting at a given material point of the aether due to the convective accelerations of the latter. The gradient part of the convective acceleration can not be observed independently from the gradient of the aether pressure φ. The only observable quantity is the last term of the acceleration. By virtue of definition (3.20) the latter adopts the form

$$F_l = \mu_0 \mathbf{A} \times \mathbf{B} . \qquad (3.25)$$

This expression is proportional to the Lorentz force experienced by a moving charge. Although the exact coefficient of proportionality can be checked only after the notion of charge is incorporated into the model, yet the expression (3.25) has an important bearing. It points out the direction in which the governing equations of the electromagnetodynamics can be generalized to be *Galilean* invariant. Note that the Maxwell equations can not be rendered Galilean invariant, but rather the "original" equations (2.6).

3.3. Estimating the Constants of Metacontinuum

Speaking about the estimation of elastic constants of the continuum (Lamé coefficients η and λ) the first one is defined by the permittivity ϵ_0 (eq.(3.22)) and vice versa – if the speed of shear waves (*speed of light*) is known, the permittivity is easily defined. However, for estimating the second Lamé coefficient λ, some completely different experiments are to be devised.

4. Manifestation of "Underdeveloped" Dimensions

Let us examine now the consequences of the presence of additional spatial dimensions of the metacontinuum which can not be identified in shear–wave experiments.

It is instructive to recall here the idea put forward by Hinton[10] in an apparent attempt to reconcile the negative results of the nineteenth–century interferometry experiments, e.g.[16], and the notion of everpervading aether which is not entrained by the gross bodies. His reasoning was that the $3^{\mathcal{D}}$ world is very thin in the fourth dimension layer which lies on the surface of $4^{\mathcal{D}}$ continuum which he identified as *aether*. According to him the $3^{\mathcal{D}}$ objects freely glide over the surface of *aether*.

Further on Hinton speculated that the thickness in the 4th dimension of what he called "material world" is so small that it cannot be perceived by our senses.[§] Carrying on with the Hinton's idea we ask the question: "What kind of manifestation is to be expected of the fact that the material world is a thin $3^{\mathcal{D}}$ layer in the $4^{\mathcal{D}}$ geometrical space?"

Our main assumption is that the presence of additional "underdeveloped" (in the sense of Hinton) dimension(s) should result into additional number of observable quantities besides those that were enough to make a self–consistent picture in the Maxwellian framework. Depending on the topology of the metacontinuum there may be (or may not be) a number of these manifestations, connected with different spins in multi–dimensional space. What is sure, however, that the existence of at least one of these additional variables is inevitable, namely of the variable which reflects the amplitude of deflection of the $N^{\mathcal{D}}$ layer in direction of the $N+1$–st dimension. In the present Section we derive the governing equation for the amplitude of the $(N+1)^{\mathcal{D}}$ deflection for the case of geometrically nonlinear very thin (virtually $N^{\mathcal{D}}$) elastic layers in $N+1$ dimensions. This equation is not a follow up from the governing equations in N dimension, just like in the real world the Schrödinger equation is not a corollary form the Maxwell equations.

Following the general framework from the previous sections we consider a metacontinuum whose dilational Lamé coefficient is (asymptotically) much greater than the shear one, i.e., $\lambda \gg \eta$. As it will be seen in what follows, the standard shell theory is not sufficient for describing the object under consideration and in order to distinguish them from the classical shallow shells we call this kind of elastic layers *gossamers*.

The thickness of the gossamer so small that the material nonlinearity of medium becomes irrelevant and the only source of nonlinearity remains the geometry. For this reason, in the present section we derive anew the shell equations with a special emphasis on the terms connected with the curvature of the middle surface.

4.1. Geometry of Shell Space. The Gossamer

Consider a N dimensional "thin" shell (called furthermore $N^{\mathcal{D}}$ *shell*) spanning a thin layer in the $(N+1)^{\mathcal{D}}$ space. Assume that the middle surface is parameterized by the N curvilinear coordinates $\xi^\alpha(\alpha = 1,\ldots,N)$. Without loosing the generality we

§ Ideologically akin was the idea of Kalutza[11] and Klein[12]. However, in these works it has been implemented in the framework of the GTO which precludes Galilean invariance and any kind of continuum–mechanics models.

assume these to be natural coordinates associated with the main curvature lines on the surface. The middle $N^{\mathcal{D}}$ surface is characterized by the first $g_{\alpha\beta}$ and second $b_{\alpha\beta}$ fundamental forms. The fundamental tensor of the $(N+1)^{\mathcal{D}}$ *space* enclosed in the $N^{\mathcal{D}}$ *shell* is defined as (see[17,8])

$$G_{\alpha\beta} = g_{\alpha\beta} - 2sb_{\alpha\beta} + s^2 c_{\alpha\beta}, \qquad G_{N+1,N+1} = 1, \qquad G_{\alpha,N+1} = 0. \tag{4.1}$$

where $s \equiv \xi^{N+1}$ is the normal to the middle surface coordinate and $c_{\alpha\beta} = b_{\alpha\delta}b^{\delta}_{\beta}$ is the third fundamental form. Here and henceforth the Greek indices range from 1 to N and serve to mark the variables in the $N^{\mathcal{D}}$shell. Italics are used for indices when the $N+1$–st dimension is concerned.

Within the order of approximation $o(s^2)$ the contravariant components of the fundamental tensor are given by

$$G^{\alpha\beta} = g^{\alpha\beta}(\xi^1,\cdots,\xi^{N-1}) + 2sb^{\alpha\beta}(\xi^1,\cdots,\xi^{N-1}) + 3s^2c^{\alpha\beta}(\xi^1,\cdots,\xi^{N-1}) + o(s^2)\ , \tag{4.2}$$

$$G^{NN} = 1, \qquad G^{\alpha N} = 0\ . \tag{4.3}$$

4.2. Covariant Differentiation in the Shell Space

This section leans heavily on Ref.[17,8] but not all of the necessary formulas are available in literature. In order to make the present paper self contained we develop here the missing relevant formulas.

The covariant derivative of a vector or two–valent tensor in $(N+1)^{\mathcal{D}}$ space is given by

$$A^n\|_i = \frac{\partial A^n}{\partial \xi^i} + \Gamma^m_{ik}A^n\ , \qquad A^{nm}\|_i = \frac{\partial A^{mn}}{\partial \xi^i} + \Gamma^m_{ik}A^{kn} + \Gamma^n_{ik}A^{mk} \tag{4.4}$$

where $\Gamma_{ij,l}$ is the covariant Christoffel symbol in N dimensions

$$\Gamma_{ij,l} = \frac{1}{2}\left(\frac{\partial G_{jl}}{\partial x^i} + \frac{\partial G_{il}}{\partial x^j} - \frac{\partial G_{ij}}{\partial x^l}\right)\ , \qquad \Gamma^k_{ij} = G^{kl}\Gamma_{ij,l}\ .$$

The contravariant symbols are obtained form the covariant through the procedure of "elevation" ("contraction") of indices.

Using the definition of fundamental tensor (4.1), one shows that a Christoffel symbol vanishes if it contains the index $N+1$ at least in two positions, i.e.,

$$\Gamma_{\alpha N,N} = \Gamma_{NN,\alpha} = \Gamma_{NN,N} = 0\ , \quad \Gamma^N_{\alpha N} = \Gamma^\alpha_{NN} = \Gamma^N_{NN} = 0\ ,$$

$$\text{for} \quad \alpha = 1,\ldots,N\ .$$

Moreover, for the symbols containing the index $N+1$ only in one position we have:

$$\Gamma_{\alpha\beta,N} \equiv -\Gamma_{\beta N,\alpha} = -\frac{1}{2}\frac{\partial G_{\alpha\beta}}{\partial s} = b_{\alpha\beta} - sc_{\alpha\beta}\ ,$$

and due to the property $G^{Nj} = \delta^{Nj}$ of the contravariant fundamental tensor:

$$\Gamma^{N+1}_{\alpha\beta} = G^{N+1,j}\Gamma_{\alpha\beta,j} = \Gamma_{\alpha\beta,N} = b_{\alpha\beta} - sc_{\alpha\beta} \ ,$$

Respectively,

$$\begin{aligned}\Gamma^{\alpha}_{\beta,N+1} = \frac{1}{2}G^{\alpha\kappa}\frac{\partial G_{\beta\kappa}}{\partial s} &= -(g^{\alpha\kappa} + 2sb_{\alpha\kappa} + 3s^2c_{\alpha\kappa})(b_{\beta\kappa} - sc_{\beta\kappa}) \\ &= -b^{\alpha}_{\beta} + sc^{\alpha}_{\beta} - s^2c^{\alpha\kappa}b_{\beta\kappa} \ ,\end{aligned}$$

Finally, for the Christoffel symbols which do not contain the index $N+1$ we derive

$$\Gamma_{\beta\gamma,\alpha} = l[g]_{\beta\gamma,\alpha} - 2sl[b]_{\beta\gamma,\alpha} + \frac{s^2}{2}l[c]_{\beta\gamma,\alpha}$$

where

$$\begin{aligned} l[g]_{\beta\gamma,\alpha} &\equiv \frac{1}{2}\left(\frac{\partial g_{\beta\alpha}}{\partial x^{\gamma}} + \frac{\partial g_{\gamma\alpha}}{\partial x^{\beta}} - \frac{\partial g_{\beta\gamma}}{\partial x^{\alpha}}\right) \\ l[b]_{\beta\gamma,\alpha} &\equiv \frac{1}{2}\left(\frac{\partial b_{\beta\alpha}}{\partial x^{\gamma}} + \frac{\partial b_{\gamma\alpha}}{\partial x^{\beta}} - \frac{\partial b_{\beta\gamma}}{\partial x^{\alpha}}\right) \qquad (4.5) \\ l[c]_{\beta\gamma,\alpha} &\equiv \frac{1}{2}\left(\frac{\partial c_{\beta\alpha}}{\partial x^{\gamma}} + \frac{\partial c_{\gamma\alpha}}{\partial x^{\beta}} - \frac{\partial c_{\beta\gamma}}{\partial x^{\alpha}}\right) \end{aligned}$$

are the connections due to the tensors $g_{\alpha\beta}, b_{\alpha\beta}$ and $c_{\alpha\beta}$, respectively. One sees that due to the curvature of the middle surface of shell the connections in the shell space are more complicated making its restriction to the $N^{\mathcal{D}}$ surface a non–Riemannian space. Note that the first term of the connections, namely $l[g]_{\beta\gamma,\alpha}$ is nothing else but the Riemannian connection ($N^{\mathcal{D}}$ Christoffel symbol) for the $N^{\mathcal{D}}$ space of the middle surface of the gossamer.

The related contravariant Christoffel symbol is expressed as usual

$$\Gamma^{\alpha}_{\beta\gamma} = G^{\alpha\kappa}\Gamma_{\beta\gamma,\kappa} = (q^{\alpha\kappa} + 2sb^{\alpha\kappa} + 3s^2c^{\alpha\kappa})(l[g]_{\beta\gamma,\alpha} - 2sl[b]_{\beta\gamma,\alpha} + \frac{s^2}{2}l[c]_{\beta\gamma,\alpha}) \ ,$$

Then

$$\Gamma^{\alpha}_{\beta\gamma} = l[g]^{\alpha}_{\beta\gamma} + 2sl[b]^{\alpha}_{\beta\gamma} + s^2l[c]^{\alpha}_{\beta\gamma}$$

where

$$l[g]^{\alpha}_{\beta\gamma} = g^{\alpha\kappa}l[g]_{\beta\gamma,\kappa} \ , \quad l[b]^{\alpha}_{\beta\gamma} = b^{\alpha\kappa}l[g]_{\beta\gamma,\kappa} - g^{\alpha\kappa}l[b]_{\beta\gamma,\kappa}$$

$$l[c]^{\alpha}_{\beta\gamma} = +g^{\alpha\kappa}l[c]_{\beta\gamma,\kappa} - 4g^{\alpha\kappa}l[b]_{\beta\gamma,\kappa} + 3c^{\alpha\kappa}l[g]_{\beta\gamma,\kappa}$$

Now we are equipped to derive the expressions for the $(N+1)^{\mathcal{D}}$ covariant derivatives $\|_i$ for the space inside the shell. By definition we have

$$A^m\|_i = \frac{\partial A^m}{\partial \xi^i} + \Gamma^m_{in}A^n \ .$$

Let us also introduce the notation

$$A^{\mu}|_{\alpha} = \frac{\partial A^{\mu}}{\partial \xi^{\alpha}} + l[g]^{\mu}_{\alpha\nu}A^{\nu} \ , \qquad (4.6)$$

which coincides with the $N^{\mathcal{D}}$ covariant derivative in the points that lie in the middle surface of the shell ($s = 0$). The only variables (4.6) that depend on the normal coordinate s are the components of the vector A^{μ}.

Acknowledging the formulas for the Christoffel symbols one derives the following expressions for the covariant derivative $\|_i$

$$A^{\mu}\|_{\alpha} = A^{\mu}|_{\alpha} + (2sl[b]^{\mu}_{\nu\alpha} + s^2l[c]^{\mu}_{\nu\alpha})A^{\nu} - (b^{\mu}_{\alpha} - sc^{\mu}_{\alpha} + s^2c^{\mu\kappa}b_{\alpha\kappa})A^{N+1}$$

It is a generalization of the respective Neuber's formula because we allow dependence on s of the components of the vector that is differentiated. Further,

$$A^{N+1}\|_{\alpha} = \frac{\partial A^{N+1}}{\partial \xi^{\alpha}} + (b_{\nu\alpha} - sc_{\nu\alpha})A^{\nu}$$

Here one should be reminded that the component A^{N+1} behaves as a scalar, as far as the subspace of the middle surface is concerned. For this reason we have

$$A^{N+1}|_{\alpha} \equiv \frac{\partial A^{N+1}}{\partial \xi^{\alpha}} \ .$$

In the same manner is obtained

$$A^{\alpha}\|_{N+1} = \frac{\partial A^{\alpha}}{\partial s} - (b^{\alpha}_{\mu} - sc^{\alpha}_{\mu} + s^2c^{\alpha\kappa}b_{\nu\kappa})A^{\nu} \ .$$

Finally,

$$A^{N+1}\|_{N+1} = \frac{\partial A^{N+1}}{\partial s} \ .$$

We also present the formulas for the covariant differentiation of tensors. Let us note again that our derivations are not restricted (as it is the case with[17] and[8]) to the middle surface but are valid for the entire shell space

$$\begin{aligned} A^{\alpha\beta}\|_{\gamma} &= A^{\alpha\beta}|_{\gamma} + (2sl[b]^{\alpha}_{\nu\gamma} + s^2l[c]^{\alpha}_{\nu\gamma})A^{\nu\beta} + (2sl[b]^{\beta}_{\nu\gamma} + s^2l[c]^{\beta}_{\nu\gamma})A^{\alpha\nu} \\ &\quad -(b^{\alpha}_{\gamma} - sc^{\alpha}_{\gamma} + s^2c^{\alpha\kappa}b_{\gamma\kappa})A^{N+1,\beta} - (b^{\beta}_{\gamma} - sc^{\beta}_{\gamma} + s^2c^{\beta\kappa}b_{\gamma\kappa})A^{\alpha,N+1} \ , \\ A^{\alpha,N+1}\|_{\gamma} &= A^{\alpha,N+1}|_{\gamma} + (2sl[b]^{\alpha}_{\nu\gamma} + s^2l[c]^{\alpha}_{\nu\gamma})A^{\nu,N+1} \end{aligned}$$

$$+(b_{\nu\gamma} - sc_{\nu\gamma})A^{\alpha\nu} - (b^{\alpha}_{\gamma} - sc^{\alpha}_{\gamma} + s^2c^{\alpha\kappa}b_{\gamma\kappa})A^{N+1,N+1} ,$$

$$A^{N+1,\beta}\|_{\gamma} = A^{N+1,\beta}|_{\gamma} + (2sl[b]^{\beta}_{\nu\gamma} + s^2l[c]^{\beta}_{\nu\gamma})A^{N+1\nu}$$

$$+(b_{\nu\gamma} - sc_{\nu\gamma})A^{\nu\beta} - (b^{\beta}_{\gamma} - sc^{\beta}_{\gamma} + s^2c^{\beta\kappa}b_{\gamma\kappa})A^{N+1,N+1} ,$$

$$A^{N+1,N+1}\|_{\gamma} = A^{N+1,N+1}|_{\gamma} + (b_{\nu\gamma} - sc_{\nu\gamma})A^{N+1,\nu} + (b_{\nu\gamma} - sc_{\nu\gamma})A^{\nu,N+1}$$

At the end we consider $A^{N+1,\beta}$ and $A^{\alpha,N+1}$ which are in fact components of a vector as far as differentiation in the middle surface of the shell is concerned:

$$A^{\alpha,N+1}\|_{N+1} = \frac{\partial A^{\alpha,N+1}}{\partial s} - (b^{\alpha}_{\nu} - sc^{\alpha}_{\nu} + s^2c^{\alpha\kappa}b_{\nu\kappa})A^{\nu,N+1} .$$

$$A^{N\beta}\|_{N+1} = \frac{\partial A^{N+1,\beta}}{\partial s} - (b^{\beta}_{\mu} - sc^{\beta}_{\mu} + s^2c^{\beta\kappa}b_{\nu\kappa})A^{N+1,\mu} .$$

$$A^{N+1,N+1}\|_{N+1} = \frac{\partial A^{N+1,N+1}}{\partial s} .$$

4.3. Governing Equations for a $N^{\mathcal{D}}$ Shell in Cauchy Form

In this section we assume that *the conservation laws are valid for the* $(N+1)^{\mathcal{D}}$ *continuum in the same tensoral form (Cauchy form) as for the three dimensional continuum.*¶Although this is a natural assumption it can not be derived strictly from $N^{\mathcal{D}}$ experience and hence it is a conjecture. It is in fact a postulate about the meta–rheology whose validity can be assessed only after the observable physical laws in $N^{\mathcal{D}}$ are recovered as corollaries.

We write the Cauchy form as follows

$$[\rho_* a^j - P^{ij}\|_i]\mathbf{g}_j = 0, \qquad i,j = 1,\ldots,N , \tag{4.7}$$

where ρ_* is the density of the $(N+1)^{\mathcal{D}}$ continuous media filling the $N^{\mathcal{D}}$ shell. It is important to note that ρ_* should not to be confused with the density of matter! We are still to define what a matter is. Respectively $\mathbf{g}_j$ are the orts of the curvilinear coordinate system; P^{ij} are the components of stress tensor; a^j are the components of the acceleration vector in the $(N+1)^{\mathcal{D}}$ space. Respectively, $\|_i$ stands for the covariant derivative in $(N+1)^{\mathcal{D}}$ space. We do not consider here $(N+1)^{\mathcal{D}}$ body forces.

Upon substituting the above defined expressions for $\|_i$ in terms of $N^{\mathcal{D}}$ covariant derivatives $|_{\alpha}$, the Cauchy law (4.7) is recast into a system for the "surface" components and a scalar equation for the $N+1$–st component, namely

$$\rho_* a^{\alpha} - P^{\beta\alpha}|_{\beta} = \frac{\partial P^{N+1,\alpha}}{\partial s} - (b^{\beta}_{\beta} - sc^{\beta}_{\beta} + s^2c^{\beta\kappa}b_{\beta\kappa})P^{N+1,\alpha} \tag{4.8}$$

¶Note that the Cauchy form only involves the operation *div* that is easily generalized to any number of dimensions.

$$+ \; 2(2sl[b]^{\alpha}_{\gamma\nu} + s^2l[c]^{\alpha}_{\gamma\nu})P^{\nu\beta} - 2(b^{\alpha}_{\nu} - sc^{\alpha}_{\nu} + s^2c^{\alpha\kappa}b_{\nu\kappa})P^{N\nu}[0.3cm]$$

$$\rho_* a^{N+1} - P^{\beta,N+1}|_{\beta} = \frac{\partial P^{N+1,N+1}}{\partial s} + (b_{\beta\nu} - sc_{\beta\nu})P^{\beta\nu} \tag{4.9}$$

$$+ \; (2sl[b]^{\beta}_{\beta\nu} + s^2l[c]^{\beta}_{\beta\nu})P^{\nu,N+1} - (b^{\beta}_{\beta} - sc^{\beta}_{\beta} + s^2c^{\beta\kappa}b_{\beta\kappa})P^{N+1,N+1}$$

These equations are to be averaged (integrated) with respect to the normal variable within the surfaces of the shell (the integrals in what follows are understood as definite integrals between $-h/2$ and $h/2$). In doing this one should be reminded that b, c, are functions of the "surface" coordinates only. For the sake of convenience we introduce the notations

$$q^{\alpha} = \frac{1}{h}\int P^{N+1,\alpha}ds , \quad \sigma^{\alpha\beta} = \frac{1}{h}\int P^{\alpha\beta}ds , \quad m^{\alpha\beta} = \frac{1}{h}\int sP^{\alpha\beta}ds,$$

$$\varphi^{\alpha} = \frac{1}{h}\int a^{\alpha}ds , \quad \psi^{\alpha} = \frac{1}{h}\int sa^{\alpha}ds , \quad \rho_* V = \frac{1}{h}(P^{N+1,N+1}_{up} - P^{N+1,N+1}_{lo}),$$

where the subscripts "up" and "lo" refer to the upper and lower shell surfaces $s = h/2$ and $s = -h/2$, respectively. Here we impose one more limitation on our model assuming that there are no "tractions" exerted upon the shell surfaces from the two adjacent $(N+1)^{\mathcal{D}}$ spaces, i.e. $P^{N+1,\beta}_{up} = P^{N+1,\beta}_{lo} = 0$. The case of nontrivial tractions needs special attention and shall be discussed elsewhere in the due extent. Then (4.9), (4.10) give

$$\rho_*\varphi^{\alpha} - \nabla_{\beta}\sigma^{\alpha\beta} = -2b^{\alpha}_{\beta}q^{\beta} - b^{\beta}_{\beta}q^{\alpha}, \tag{4.10}$$

$$\rho_*\varphi^{N+1} - \nabla_{\beta}q^{\beta} = b_{\beta\nu}\sigma^{\beta\nu} - c_{\beta\nu}m^{\beta\nu} + \rho_* V \tag{4.11}$$

The system (4.10), (4.11) is coupled by means of the "momentum–of–impulses" which can be derived from (4.10) when multiplied by s and integrated across the shell, namely

$$-\rho_*\psi^{\alpha} + \nabla_{\beta}m^{\alpha\beta} = q^{\alpha}. \tag{4.12}$$

The quantity q^{α} can be excluded due to the last relation (upon neglecting the moments of the inertia ψ^{α}) in order to obtain

$$\rho_*\varphi^{\alpha} = \nabla_{\beta}\sigma^{\alpha\beta} - 2b^{\alpha}_{\beta}\nabla_{\nu}m^{\beta\nu} - b^{\beta}_{\beta}m^{\alpha\nu}, \tag{4.13}$$

$$\rho_*\varphi^{N+1} = \nabla_{\beta}\nabla_{\nu}m^{\beta\nu} + b_{\beta\nu}\sigma^{\beta\nu} - c_{\beta\nu}m^{\beta\nu} + \rho_* V \tag{4.14}$$

Eqs.(4.13)–(4.14) present the Cauchy form for the geometrically nonlinear shell equations which system is the basis of the considerations to follow.

Here the notion of the geometrization of physics becomes transparent. If the observer is confided to $N^{\mathcal{D}}$ space of the middle–surface he will appreciate the presence of the $N+1$–st dimension as additional terms in balance law (4.13), (4.14) which terms are not present in the Cauchy form for the $N^{\mathcal{D}}$ continuous media. The said terms are proportional to the different curvature forms and this fact is the quantitative expression of Riemann[21]–Clifford[7] idea that the physical laws are a manifestation of the deformation of the geometrical space. In our opinion, the notion of a "geometrical" space does not make much sense in itself, without assuming that some material medium is filling the geometrical space. That is why we do not consider here a mere geometrical space but a mechanical structure (N^D shell) whose deflections in higher-dimensional space result into apparent additional forces in the middle-surface.

4.4. Elastic Shell with Momentum Stresses

According to the Kirchhoff-Love hypothesis, the displacements u_α in the shell space are related to the N^D displacements $\tilde{u}_\alpha$ in the shell middle surface as follows

$$u_\alpha = \tilde{u}_\alpha - s\nabla_\alpha\zeta, \qquad u_4 = \zeta \ , \tag{4.15}$$

where ζ stands for the shape function of deformation (deflection) of the middle surface of shell in direction of $N+1$–st dimension.

After some manipulations we obtain from (4.15) that

$$\varphi_\mu = \frac{\partial^2 \tilde{u}_\mu}{\partial t^2}, \quad \psi_\mu = -\frac{h^2}{12}\frac{\partial^3\zeta}{\partial t^2 \partial\xi^\mu}, \quad \varphi_{N+1} = \frac{\partial^2\zeta}{\partial t^2}, \tag{4.16}$$

Here it becomes evident that the moments of the acceleration ψ_μ are of second order and that justifies neglecting them above. In terms of coordinates that are measured along the arcs of the middle surface, the second fundamental form adopts the following simple form

$$b_{\alpha\beta} = -\nabla_\alpha\nabla_\beta\zeta \ . \tag{4.17}$$

Note that for coordinates, generally not coinciding with the arks of the main curvature lines, the above expression of the second fundamental form involves nonlinear terms.

It is time now to couple the Cauchy equations with constitutive relations. Unlike the Cauchy form, the full nonlinear constitutive relations can not be derived in the Eulerian framework. It goes well beyond the framework of the present work to derive them in full detail, especially as far as the laminar components are concerned. We resort here to **linear** constitutive relations in the form (see[9])

$$\sigma^{\alpha\beta} = (\lambda+\eta)g^{\alpha\beta}(\nabla_\nu\tilde{u}^\nu) + \eta\nabla^\beta u^\alpha \ , \tag{4.18}$$

$$m^{\alpha\beta} = -D\nabla^\alpha\nabla^\beta\zeta \ . \tag{4.19}$$

where D is called stiffness of shell. We have abandoned the full derivation of constitutive relations from the respective $(N+1)^D$ relations, but the latter can be used in a heuristic manner to assess at least the qualitative outlook of the expression for the stiffness D, namely

$$D = (\lambda+\eta)\frac{h^2}{12} \ . \tag{4.20}$$

The last formula is very important for our conjectures about shells of material which has a very large dilational modulus. Although in (4.20) the stiffness D is proportional to h^2, it can not be neglected because it contains also the dilational Lame coefficient λ whose ratio to the shear Lame coefficient η is an independent **large** parameter. This requires keeping in the equation the terms proportional to D. Then upon introducing (4.18), (4.19) in Cauchy equations we get for the laminar components:

$$\begin{aligned} \rho_*\frac{\partial^2\tilde{u}^\beta}{\partial t^2} &= (\lambda+\eta)\nabla^\beta(\nabla_\nu\tilde{u}^\nu) + 2\eta\Delta u^\beta \\ &- (\lambda+2\eta)\frac{h^2}{12}[2\nabla^\beta(\nabla\zeta\cdot\nabla(\Delta\zeta)) + (\Delta\zeta)\nabla^\beta\Delta\zeta] \end{aligned} \tag{4.21}$$

where a term proportional to $\rho_* h^2$ has already been neglected because the density of shell material is not considered as a large parameter. Here we do not consider the case of nontrivial tractions on the shell surfaces which requires additional assumptions about their asymptotic order. We keep, however, the term responsible for the normal pressure (parameter V in the equation to follow). Then for the amplitude ζ of normal deflection in direction of $N+1$–st dimension we derive

$$\rho_*\frac{\partial^2\zeta}{\partial t^2} = \rho_* V + D\left[-\Delta\Delta\zeta + (\nabla_\beta\nabla_\delta\zeta)(\nabla^\beta\nabla_\mu\zeta)(\nabla^\mu\nabla^\delta\zeta)\right] - \sigma^{\beta\alpha}(\nabla_\beta\nabla_\alpha\zeta) \ , \tag{4.22}$$

where $\Delta \equiv \nabla_\nu\nabla^\nu$, $\Delta\Delta \equiv \nabla_\nu\nabla^\nu(\nabla_\kappa\nabla^\kappa)$. Once again terms proportional to $\rho_* h^2$ are neglected with respect to those proportional to ρ_*.

4.5. The Implication of Very Large Dilational Elasticity Coefficient

The fact that the dilational Lamé coefficient λ of *gossamer* is much larger than the shear coefficient η offers an opportunity to drastically simplify the model of meta-continuum. Indeed if we develop with respect to the powers of the small parameter $\varepsilon = \eta/\lambda$ we get from (4.21) that

$$\nabla^\beta\nabla_\nu u^\nu = 0 + O(\frac{\eta}{\lambda}) \implies \nabla_\nu u^\nu = \kappa_0 = \text{const} \implies \sigma^{\alpha\beta} = \sigma_0 g^{\alpha\beta} \ , \ \sigma_0 = \eta\kappa_0 \ ,$$

where κ_0 is dimensionless divergence of the displacement field in the middle surface. $k_0 > 0$ means uniform dilation of the middle surface, while $k_0 < 0$ reflects the case of uniform compression. Thus, in the lowest asymptotic order of ε we are faced with uniform compression/dilation in the middle surface of *gossamer* and with constant membrane stress σ acting in the middle surface. This allows to effectively decouple the laminar deformations u^α from the deflection ζ and the term in (4.22) containing $\sigma^{\alpha\beta}$ becomes simply $\sigma_0\Delta\zeta$.

4.6. The Boussinesq Equation

The only term which looks unusual in (4.22) is the cubic nonlinear term. However, in dimension one the equation under consideration is exactly the cubic $1^{\mathcal{D}}$ nonlinear Boussinesq equation. Guided by the desire to benefit from the vast knowledge accumulated for equations of type of Boussinesq, we replace in a paradigmatic way the cubic term in (4.22) by $(\Delta\zeta)^3$. It is beyond doubt that the qualitative behaviour of the solutions will be quite similar. However, the extent to which they will be quantitatively close as well, remains a heuristic assumption. Then we arrive at

$$\rho_* \frac{\partial^2 \zeta}{\partial t^2} = -\rho_* V + D\left[-\Delta\Delta\zeta + (\Delta\zeta)^3\right] - \sigma_0 \Delta\zeta \,, \tag{4.23}$$

Here the negative sign of term V allows us to use positive values of V when it acts as a compression. We render the last equation dimensionless by introducing the scales

$$\zeta = L\zeta' \,, \quad \mathbf{x} = L\mathbf{x}' \,, \quad t = \frac{L}{c_f} t' \,, \quad c_f \equiv \sqrt{\frac{\sigma_0}{\rho_*}} \,, \tag{4.24}$$

Here c_f has dimension of velocity. In fact it is proportional to the speed of light. Note that the scale for ζ and the length scale of the localized wave coincide (the length L). This is imposed by the balance between the nonlinearity and dispersion. Only this kind of scaling allows us to have both effects in the equation. Finally, the dimensionless form of the wave equation of *gossamer* reads

$$\frac{\partial^2 \zeta'}{\partial t^2} = -V' + \beta\left[-\Delta\Delta\zeta' + (\Delta\zeta')^3\right] + \epsilon\Delta\zeta' \,, \tag{4.25}$$

where $\beta = D\sigma_0^{-1}L^{-2}$, $\epsilon \equiv sign[k_0]$ and V' is the dimensionless value of the hydrostatic pressure. The primes denoting dimensionless variables are henceforth omitted without fear of confusion. Eq.(4.25) is our "master" wave–mechanics equation.

Now $\varrho\beta$ is the only nondimensional parameter and if it is significant, then without loosing the generality it may be set equal to zero. This defines the length scale L of the wave–particles as

$$L \sim \sqrt{\frac{D}{\sigma_0}} \sim h\sqrt{\frac{\lambda}{\eta}} \,,$$

where h is the thickness of *gossamer*. It has been assumed that *gossamer* is extremely thin, but on the other hand it is also very still $\lambda \gg \eta$. The interplay between these two effects define the length scale L of wave–particles which will be of different asymptotic order than the thickness h. Thus we secure that the equations of *gossamer* are used in the region of their applicability $L \gg h$.

5. Models for Loading the Metacontinuum

There are different ways to create uniform membrane tension in the middle surface of *gossamer*. One can consider an initially planar sheet of *gossamer* undergoing

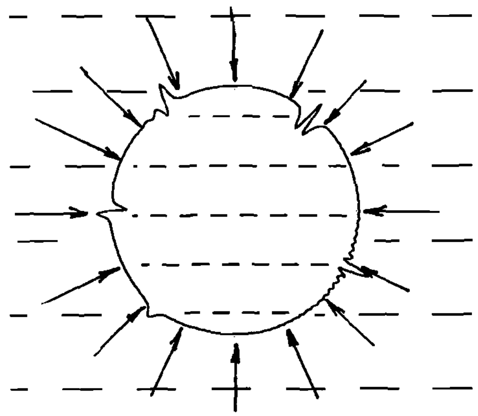

Figure 1: The bubble–Universe and its Loading

uniform dilation/compression at its rims, but then there will be some inhomogeneity near the rims which will make the shell approximation unacceptable. Another way to achieve the same result is to consider a large $N^{\mathcal{D}}$ bubble (hypersphere) of *gossamer* subjected to hydrostatic pressure from the adjacent $(N+1)^{\mathcal{D}}$ spaces. Three distinct cases are discerned: the bubble is compressed from the outside $(N+1)^{\mathcal{D}}$ (positive hydrostatic pressure, negative membrane stress σ_0 in the middle surface); the bubble is inflated from the enclosed $(N+1)^{\mathcal{D}}$ space (negative pressure, positive membrane stress); the hydrostatic pressure is equal to zero.

5.1. Model I: Uniform Compression

Consider now the first case when the bubble is compressed from outside (Fig. 1). This is a well known situation of spherical shell under hydrostatic pressure for which a buckling phenomena is known to take place giving birth to localized, etc. corrugations of the middle surface of *gossamer* through bifurcation of the motionless state.

The motionless (equilibrium) state is characterized by the balance between the membrane tension and the hydrostatic compression, namely

$$\sigma_0 \frac{N}{R} = -V \quad \Longrightarrow \quad \sigma_0 < 0 \,.$$

Here R is the radius of the bubble, N dimension of the middle surface of *gossamer*

It is clear that the force V must be small enough (of order of the inverse radius of Universe). In the middle surface of the gossamer, it creates a uniform longitudinal tension σ_0. We introduce the relative displacement $\bar{\zeta}$ according to the formula

$$\zeta = R^{-1} + \bar{\zeta}$$

and rewrite (4.22) as follows (be reminded that the coordinates are presumed to be [illegible] and to coincide with the main curvature lines of the bubble),

$$\frac{\partial^2 \zeta}{\partial t^2} = -c_f^2 \Delta\bar{\zeta} + \beta[(\Delta\bar{\zeta})^3 - \Delta\Delta\bar{\zeta}], \tag{5.26}$$

5.3. *Model II: Uniform Dilation*

The other case is to have uniform dilation. This can be achieved through inflating hydrostatic pressure acting from the inside of the bubble. Then the rest state defines the membrane tension is absolutely the same manner as for the compression with the only difference that now the sign of σ_0 is the opposite. If $\sigma_0 = 0$, the situation is very similar, because for $\sigma_0 \geq 0$ there is no buckling.

$$\frac{\partial^2 \zeta}{\partial t^2} = \Delta\zeta + \beta\left[-\Delta\Delta\zeta + (\Delta\zeta)^3\right], \tag{5.27}$$

where the dimensionless variables are introduced in the same manner. As earlier shown, the parameter c_f has dimension of velocity. Note that the value of c_f does not necessarily coincide with the speed of light.

The scales for time and length from (4.24) are defined so as to keep all of the effects in (5.26). It shall be discussed bellow that the membrane tension can be defined from the Newton's gravitational constant. Then the comparison of L with the measures of elementary particles can give us some qualitative assessment for the shell stiffness D.

It is well known form the theory of KdV solitons that in order to have a soliton of a given amplitude (or *mass*) one has to "invest" in the initial moment certain strictly defined quantity of energy. This poses some ideological inconvenience with *model II* in the sense that someone must create in the initial moment of time the whole ensemble of particles, waves, etc. (the *cosmos*) exerting deliberate pushes at certain points of the bubble–Universe. Then the total energy expended in the initial moment defines the energy capacity of the Universe since no energy could be created or destroyed due to the conservation properties of the Boussinesq equation. Yet, leaving apart the ideological matters we shall consider *model II* on equal footing with *model I*. Moreover that a vast body of literature is devoted to soliton problems that are described by *model II* rather than *model I*.

6. The Nonlinear Eigen–Value Problems: The Solitons

6.1. *Flexural Solitons (Flexons) or The Particles*

In the case of *model I*, even the linearized equations of *gossamer* undergo bifurcation because the the external hydrostatic pressure is known the provoke buckling of shell. In the stationary case the buckling is governed by the bifurcation of solution

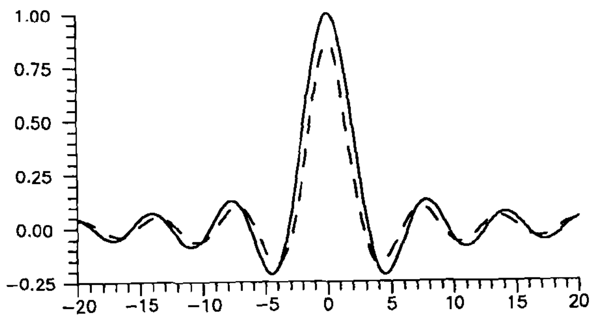

Figure 2: Flexon solution for case I: Moderate amplitude: - - - - - linear *sinc* solution scaled to have the same asymptotic behaviour as the nonlinear solution; ——— nonlinear solution.

of the following dimensionless boundary value problem in infinite domain

$$b - b^3 + \frac{1}{r^{N-1}}\frac{d}{dr}r^{N-1}\frac{d}{dr}b = 0, \quad b \to 0 \quad \text{for} \quad r \equiv |\mathbf{x}| \to \pm\infty , \tag{6.1}$$

where $b = \Delta\zeta$ is the curvature of the shape of the nontrivial transverse (or flexural) elevations (humps) on its surface. We call *particles* the solitary waves of the governing equations of gossamer. It is important to verify whether the interactions among the localized solutions of our equations are indeed elastic. One should be aware of the fact that the metacontinuum is elastic does not necessarily yields to elastic interaction of the particles[5]. Since we had not proven yet whether our solutions for the flexural localized waves live up to the name *solitons* we prefer to call them *flexons* which carries also a hint of their origin (flexural deformations or deflections).

It is known that even the linearized version of eq.(6.1) possesses in $3^{\mathcal{D}}$ along with the trivial solution a localized non–trivial one:

$$\zeta = Ar^{-1}\sin(r) , \tag{6.2}$$

where A is an arbitrary constant. It is amazing that this solution has been just recently spotted as a candidate for "single event"[1] (particle) by the specialists of de Broglie wave mechanics. The advantage of the proposed here *model I* is that it identifies also the form of nonlinear term the latter defines the amplitude of the wave. We had solved (6.1) numerically and in Fig. 2 the solution is presented by the solid line and juxtaposed to the corresponding linear solution $sinc = r^{-1}\sin(r)$ scaled to have the same asymptotic decay at infinity as the nonlinear. It goes well beyond the frame of the present note to provide full details of numerical algorithm. It is called

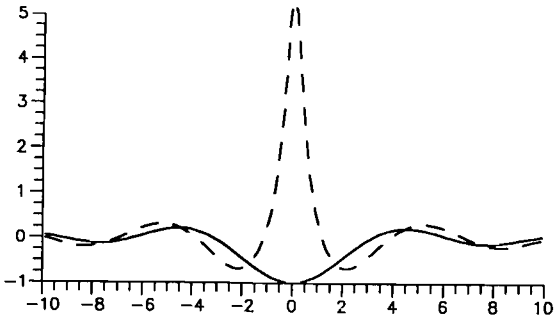

Figure 3: Flexon solution for case I. Large amplitude: ——— – linear *sinc* solution; – – nonlinear solution.

Method of Variational Imbedding (MVI) and is descendant of the algorithm proposed in[2] (see also[3] for details of implementation).

The difference between the nonlinear and linear solutions presented here is only quantitative and one can think of our solution approximately as of the *sinc* with a particular amplitude. The fact that we have obtained this *sinc*–shaped solution only for one single value of its amplitude is rather significant. We call *mass* of particle the amplitude of a *flexon*. Naturally, we call the *flexon* with negative amplitude *antiparticle*. The simplified eq.(6.1) does not distinguish between particles and antiparticles. However, in the original model of bubble–universe a slight difference between particles and antiparticles is to be expected due to the terms proportional to the curvature of the undisturbed bubble. In fact, it is natural to expect that either the particles or the antiparticles have better chances to appear as a result of the bifurcation depending on which of them minimize the stored elastic energy of the shell.

The nonlinear equation (6.1) admits more than one nontrivial solution. In Fig. 3 another solution which we have obtained numerically by the MVI is shown. It is of much larger amplitude. As it happens very often in nonlinear bifurcation problems, more than one nontrivial solution may appear for the same values of the governing parameters.

The case of inflating pressure (*model II*) is completely different. As already shown, it yields the classical Boussinesq equation which reduces in the stationary case to the following

$$-b - b^3 + \frac{1}{r^{N-1}}\frac{d}{dr}r^{N-1}\frac{d}{dr}b = 0\ , \quad b \to 0 \qquad \text{for} \quad r \equiv |\mathbf{x}| \to \pm\infty\ , \tag{6.3}$$

In dimension one this equation appears several times (twice at least) in the present book[4,13]. Then the localized solution is the well known Boussinesq *sech*. We do not

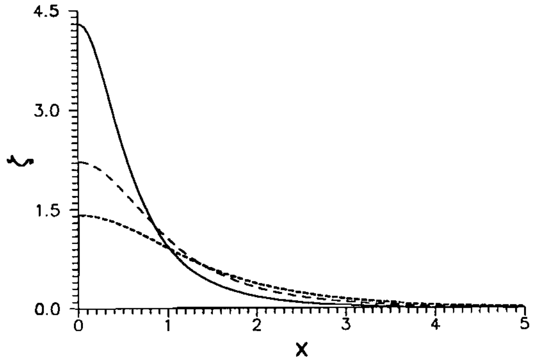

Figure 4: Flexon solution for case II: - - - - - - 1D case; – – – – 2D case; ——— – 3D case.

know the analytical expression replacing the *sech* in three dimensions, although it is very natural to expect that such a solution may very well exist in the literature. Here we would rather use the same procedure of MVI to calculate it numerically. Fig. 4 shows the respective result. Note that the numerical soliton for the 1D case (short dashes) virtually coincide with the analytical solution $\sqrt{2}\mathrm{sech}(x)$ and can not be distinguished in the figure. For comparison are shown also the numerical results for the 1D, 2D and 3D cases. The 1D–solution coincides within the numerical error with the analytical *sech* solution.

6.2. Torsion Solitons (Twistons) or The Charges

Although the way we load the gossamer is the simplest one (the hydrostatic pressure), it turns out that a complicated "cosmological" picture appears with host of different localized entities–solutions. It is interesting to mention that bifurcation and symmetry breaking does not affect just the flexural deformation. The strained state of the gossamer resembles the flow with continuously distributed sinks, provided that the displacement field in the middle surface is compared to the velocity field of the above described flow. Then due to the analogy between the Stokes flows and elastic deformations one is to expect "vortices" of the displacement field. The linearized equations do possess solutions like that but the amplitude is not specified and singularity is present in the center of the "vortex". Yet the vortex solutions are suggestive enough because of the conservation of the topological charge in a particular solution. Only the nonlinear elasticity effects for the $N^{\mathcal{D}}$ elastic medium in the middle surface can specify the amplitude of the vortex. In fact the solution we are speaking about is the "reversed" situation with the Poynting solution[20]. In Poynting solution, due to the nonlinear–elasticity effects, a wire is elongated when twisted. Here is the opposite

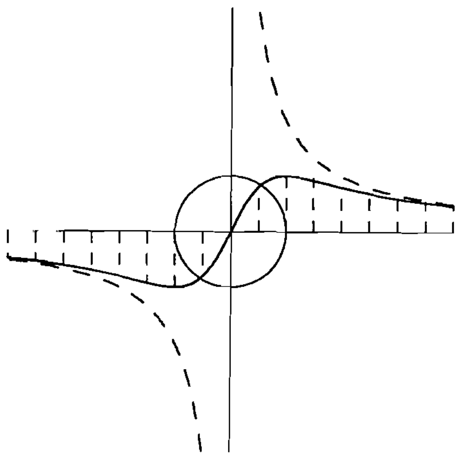

Figure 5: A qualitative picture of *twiston*.

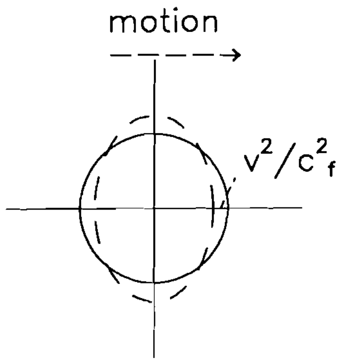

Figure 6: Sketch of the Lorentz–FitzGerald contraction of solitons: ——— - rim of particle when at rest; - - - - rim of particle moving with celerity V.

situation. The metacontinuum is spontaneously twisted when stretched (compressed).

For the time being we do not have the numerical solution (as we do for the above discussed case of the *flexon*), but some preliminary derivations for the two dimensional analogy allow us to propose the qualitative picture (see Fig. 5) of the "vortex" of displacement field which arises in the wake of the spontaneous breaking of symmetry. The problem is in fact very similar to the vortex solutions discussed in the illuminating paper[19], although in the latter a somewhat different governing equation (Ginzburg–Landau equation) is considered.

We call the localized solution of vortex type *twiston* in order to distinguish it from the fluid vortices. The "charge" of a *twiston* is nothing else but the so-called topological charge of the displacement field. It is clear then that the charge can appear only in a discrete set of magnitudes because of the discrete spectrum of the nonlinear eigen-value problem whose solution is the *twiston*. The obvious symmetry of the linearized problem hints at the conclusion that the charge can be positive or negative. Depending on their charges, two *twistons* repel or attract each other according to Coulomb's inverse-square law (just as two vortices do).

It is important to mention here that *twistons* and *flexons* are not strictly orthogonal creatures because between the longitudinal and transverse motions of a $N^{\mathcal{D}}$ shell exists a coupling. However, the quantitative effect of the latter is very weak (see eqs.(4.21), (4.22)) and they can be treated for most of the things as independent structures. Yet this coupling is enough to cause a slight elevation (depression) of the

gossamer surface in the region of localization of a *twiston*. In other words, the *twiston* has its own mass which can be either positive or negative and is much smaller than the mass of the *flexon* (the neutral particle). By analogy one can call it "mass of electron (positron)". Massive charged particles can appear when a *twiston* "nests" over a *flexon* and then the mass is the superposition of the amplitude of the neutron (*flexon*) and the amplitude of the flexural deformation associated with the *twiston* (positron or electron).

6.3. *Density Solitons*

Alongside with the flexural and torsion solitary waves, one must expect also solitary waves connected with the compressibility of metacontinuum. Unfortunately, the equation of state of metacontinuum requires additional conjectures that go beyond the scope of the present paper. For this reason a quantitative numerical solution for the density solitons can not be attempted. However, a number of qualitative conclusions can be reached on the basis of the known properties of the compression waves in solids. First of all, the speed of these waves is limited by the speed of sound of the metacontinuum. The steepness of a density wave increases with its celerity. This means that the swifter the movement of the wave – the smaller its spatial extent. On the other hand, the density solitons are expected to interact almost insignificantly with the matter (the *flexons* and *twistons*) and in this instance they resemble to a great extent the behaviour of the neutrino. The qualitative description given just above allows us to propose the conjecture that the effects of the all pervading compression motion that are orthogonal to the matter are quite similar to what the ancient School of Stoa called *Pneuma* (see, e.g.[22]).

7. Dynamics of Patterns in the Metacontinuum

7.1. *The Hamiltonian Formulation*

Consider the "master wave equation" in a form valid for both models (I and II).

$$\frac{\partial^2\zeta}{\partial t^2} = \epsilon c_f^2 \Delta\zeta + \beta\left[(\Delta\zeta)^3 - \Delta\Delta\zeta\right] , \qquad (7.1)$$

where $\epsilon = -1$ refers to *model I* (uniform compression) and $\epsilon = +1$ – to *model II* (uniform dilation) and β is dimensionless dispersion coefficient. We keep here the dimensionless notation c_f for the sake of convenience. As far as localized waves are concerned, asymptotic b.c. must be satisfied for the wave–amplitude ζ, namely

$$\zeta \to \text{const} , \quad \nabla\zeta \to 0 , \quad \Delta\zeta \to 0 , \quad \nabla\Delta\zeta \to 0 \quad \text{for} \quad |\mathbf{x}| \to \infty . \qquad (7.2)$$

For these b.c. the following Hamiltonian representation is readily derived upon multiplying (7.1) by $\Delta\zeta_t$ and integrating over the infinite domain

$$\frac{dH}{dt} = 0 \,, \quad H \equiv \frac{1}{2}\int_{-\infty}^{\infty}\left[(\nabla\zeta_t)^2 + \epsilon c_f^2(\Delta\zeta)^2 + \frac{1}{2}\beta(\Delta\zeta)^4 + \beta(\nabla\Delta\zeta)^2\right] d^N\mathbf{x} \,. \qquad (7.3)$$

Before proceeding further it is worth mentioning here that energy functional (7.3) for the *model I* $\epsilon = -1$ is not positive definite, but unlike the Boussinesq wave equation (see [24,5]) the solution will not blow–up in final time because the cubic nonlinearity dominates the linear term when the solutions starts growing. The other source of incorrectness in the sense of Hadamard can also be the improper sign of the dispersion parameter β. As far as in our model it β represents the stiffness of shell, it could never be negative (as is for instance the case with the Boussinesq equation treated in another chapter of the present book[13]). Although the improper equation with $\beta < 0$ may exhibit a rich phenomenology[13], it is hard to justify it as a model for whatever physical process.

7.2. From Metadynamics of Underlying Continuum to Dynamics of Centers of Localized Structures

The main significance of the Hamiltonian formulation is that it provides the means to build the dynamical model for the phase objects (*solitons*). If the shape of localized wave is known, the Hamiltonian dynamics for the discrete system of centers of "particles" can be derived from (7.1) with a good approximation provided that they do not interact as strongly so as to change their shapes. Let us consider the case of two *solitons*–particles. Then the wave amplitude can be decomposed as follows:

$$\zeta = F_1(\mathbf{x} - \mathbf{X}_1(t)) + F_2(\mathbf{x} - \mathbf{X}_2(t)) + F_{12}(\mathbf{x} - \mathbf{X}_1(t), \mathbf{x} - \mathbf{X}_2(t)) \,. \qquad (7.4)$$

Here F_i are the shape functions of the waves–particles and $\mathbf{X}_i(t)$ are the trajectories of their centers. Now the time derivative of each shape function can be expressed as follows

$$\frac{dF_i}{dt} = -\nabla F_i \cdot \frac{d\mathbf{X}_i}{dt} \qquad (7.5)$$

which means that the discrete Hamiltonian contains quadratic forms of the velocities of centers of the form

$$\sum_i \mathcal{A}_i \cdot \dot{\mathbf{X}}\dot{\mathbf{X}} \qquad (7.6)$$

The matrix $\mathcal{A}_i$ of quadratic form of this type is positive definite and (7.6) can be interpreted as definitive relation for a kinetic energy of the center of particle. Respectively, the term F_{12} will contribute a term which depends on the relative position of particles and plays the role of potential of interaction.

According to the Hamiltonian dynamics described here, the Euler–Lagrange equations for discrete system of centers of *solitons* give $\ddot{\mathbf{X}} = 0$. It means that without interactions or other types of forces, a *soliton* will remain in inertial motion with constant velocity.

8. Gossamer Cosmology

8.1. The Shell Membrane Tension or The Gravitation

According to the picture drawn here, the particles are localized elevations (humps) of the gossamer surface. It is clear that the amplitude of the solitary wave decays at infinity (see Fig. 2 or 4) and far from the centre of a particle the shape is smooth enough and hence one can neglect the fourth derivatives in the governing equation. In other words, the long-range interactions of the solitary waves (particles) are governed by the following equation

$$\frac{\partial^2\zeta}{\partial t^2} = \epsilon c_f^2 \Delta\zeta \qquad (8.1)$$

First we mention that for stationary particles eq.(8.1) reduces to Laplace equation and in $3^{\mathcal{D}}$, in terms of spherical coordinates the solutions far from the coordinate origin (the centre of the particle under consideration) is $\zeta = const \cdot r^{-1}$. Since the force experienced from a point of the membrane (the gossamer) is proportional to $\nabla\zeta$, it is clear that the asymptotic behaviour of the said force is r^{-2}. The force is attractive because it acts to "pull" the material points of the gossamer towards the centre of the particle under consideration. Thus we arrive at Newton's law of gravitation with gravitational constant that is proportional to the membrane tension in the $N^{\mathcal{D}}$ shell and hence to V and the radius of the Universe. The inverse-square law is a manifestation of the fact that the shell is a $3^{\mathcal{D}}$ continuum.

In fact, the attraction between particles arises out of the disturbances they introduce in the uniform membrane anti-tension acting in the shell. Thus, we discover a quantitatively reversed but philosophically identical picture to the concept of Mach that the gravitation is due to the interaction with the quiescent matter at the rim of the Universe. Here apply the words of Maxwell from the end of Part IV of[14]:

> *"The assumption, therefore, that gravitation arises from the action of surrounding medium in the way pointed out, leads us to the conclusion that every part of this medium possesses, when undisturbed, an enormous intrinsic energy, and that the presence of dense bodies influences the medium so as to diminish this energy whatever there is a resultant attraction.*
>
> *As I am unable to understand in what way a medium can possess such properties, I cannot go any further in this direction in searching for the cause of gravitation."*

We can decipher here Maxwell's words "intrinsic energy of surrounding medium" as the stored in the gossamer elastic energy created from the action of the membrane tension. Indeed, the presence of humps over the gossamer surface influences the medium so as to diminish the stored energy and there arises a resultant attractive force.

For the *model II*, the words of Maxwell do not apply in such a straightforward manner.

The gravitational force for *model I* can be pointwise non–monotonous (see the shape of *flexon* presented in Fig. 2), i.e. depending on the distance between the particles they, can attract or repel each other. That is why the gravitational force will appear in this model much weaker than the electromagnetic one. In fact, it will be a mean value averaged over the different positions of the particles. It is to be expected that the total average is positive (attraction), since the positive humps of the *flexon* amplitude are larger than the negative.

8.2. Lorentz-FitzGerald Contraction

It is interesting to investigate the shape of a localized solution when it propagates with a constant celerity V alongside the axis Oz. For simplicity we consider here the linearized version of "master equation" eq.(7.1)

Acknowledging the direction of propagation of a soliton breaks the radial symmetry of the problem, i.e. one can not expect a time–dependent solution to (7.1) with a radial symmetry. One can introduce a moving frame (a new coordinate $z' = z - Vt$) and to seek for a stationary, in the moving frame, solution of (7.1),namely

$$V^2\frac{\partial^2\zeta}{\partial z^2} = \epsilon c_f^2\Delta\zeta - \beta\Delta\Delta\zeta, \tag{8.2}$$

The sought solution can be represented into series whose first two terms are

$$\zeta = \frac{\epsilon c_f^2}{\epsilon c_f^2 - V^2}\left[\zeta_0(r) + \frac{V^2}{c_f^2}\zeta_1(r)\cos(2\theta) + \ldots\right], \tag{8.3}$$

where θ is the latitude of the spherical polar coordinates with axis coinciding with the coordinate axis Oz. For the unknown coefficients $\zeta_i(r)$ are derived equations in the usual manner. The factor in the r.h.s. of (8.3) shows the increase of the *mass* of *flexon* in comparison with the rest mass (represented by function $\zeta_0(r)$). In case of *model I* ($\epsilon < 0$) the mass decreases with increase of celerity V, while for the *model II* ($\epsilon > 0$) one has the well known increase of mass with velocity (called relativistic increase of mass).

Let us define a "rim" of a particle as the value of the radial coordinate for which the local amplitude of the *flexon* is equal, say, to 0.01 of the maximal amplitude. This value is a function of the latitude and longitude of the spherical coordinates. Only for the solution with radial symmetry (6.2), the radius is constant and hence the rim of particle is spherical (see the solid curve in Fig. 6). According to (8.3) the shape of a moving particle (the cross-section of a propagating *flexon*) is more like an ellipsoid (the dashed curve in Fig. 6). The apparent contraction of the particle in direction of movement and the increase of its amplitude (its mass) are proportional to the square of the ratio V/c_f which is in qualitative agreement with the Lorentz-FitzGerald[25].

8.3. Dispersion and "Red Shift"

It is peculiar that the dispersive Boussinesq equation possesses solutions (see Figs. 2,3 of Ref.[4]) that propagate with the characteristic speed (in our case c_f) and undergo some aging ("red shift") in the sense that their support increases while the amplitude decreases. Then far from the source, one can not distinguish between the red–shifting due to dispersion or to Doppler effect if present. This means that the "red shift" is an innate feature of our model and "Big–Bang" may not be needed for explaining the red shift.

9. Conclusion

We have shown that considering a very thin $N^{\mathcal{D}}$ shell (called *gossamer*) in the $(N+1)^{\mathcal{D}}$ space one can deduce the Maxwell equations from the governing equations for laminar displacements and a dispersive nonlinear Boussinesq equation akin to Schrödinger's equation) for the amplitude of deflection alongside the $N+1$–st dimension. These equations constitute the model of *gossamer* the latter playing the role of a metacontinuum: carrier of the interactions. The localized solutions (*solitons*) of the dispersive wave equation are interpreted as particles reversing thus the de Broglie wave–particle dichotomy. The localized vortex–like solutions are identified as charges. For the case of $3^{\mathcal{D}}$ *gossamer* imbedded in a $4^{\mathcal{D}}$ space the asymptotic expression for the attraction between *solitons* is the Newton inverse square law.

Considering the matter (particles and charges) as *solitons* of the metacontinuum (field) yields a self–consistent cosmological picture. The soliton paradigm helps to explain why the classical aether (or which is the same – the metacontinuum of the present work) is not entrained by the motion of the gross matter.

We believe that the above arguments suffice to hope that if there is any resemblance of the drawn here picture to a unified field theory, it is not entirely accidental.

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DYNAMICS OF VORTICES IN NONLINEAR FIELDS

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ABSTRACT

I review a class of problems with spontaneously broken symmetry dominated by interaction and motion of defects (vortices). The approach we follow reduces the large-scale dynamics in a diluted gas of defects to a "particle–field" problem including equations of motion of defects and equations of a field that mediates their interaction. The incompatibility of the symmetries of the far and near field approximations is identified as the mechanism which sets the vortices into motion. The vortex velocity is computed as an adjustable parameter in an asymptotic procedure matching the near and far field solutions. This determines equations of motion that have to be further solved simultaneously with far field equations. I discuss applications of this approach to different physical problems, including dynamics of dislocations in striped patterns, spiral waves and vortices in superfluids, and describe additional complications due to inhomogeneity of the system, effects of mean flow, and presence of several resonantly interacting modes.

1. Introduction

The study of vortex dynamics is mainly motivated by our interest to the phenomenon of *weak turbulence* which can be understood in terms of motion and interaction of stable localized structures: *defects* in the prevailing ordered state. When defects are present, the system is disordered *globally*, though it may be *locally* ordered almost everywhere. A diluted *gas* of defects would typically evolve on a time scale *slow* compared to a characteristic time of relaxation to a locally ordered state. Evolution to the global order, mediated by the motion of defects, occurs on a much longer time scale. In cases that interest us most, global order is never achieved, and the defects either move, annihilate, and are created persistently, or freeze in a state of quenched disorder.

The weak defect-mediated disorder is similar to our physical world which has a hierarchy of weaker and stronger interactions, and allows to apply rational approximations separating motions on different scales. Were it not so, neither rational beings capable to be physicists nor physics itself would exist, and the chaos in the biblical sense of the word – strong turbulence, if you wish – would prevail.

The necessary ingredients of the theory of defect-mediated disorder is the derivation of equations of motion of defects under the action of the *far field* mediating their

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