

## Continuum Models and Discrete Systems

*Proceedings of 8<sup>th</sup> International Symposium, June 11-16, 1995, Varna, Bulgaria, ed. K.Z. Markov  
© World Scientific Publishing Company, 1996, pp. 370-394*

---

## DISCRETE OUT OF CONTINUOUS: DYNAMICS OF PHASE PATTERNS IN CONTINUA

C. I. CHRISTOV<sup>1</sup>

*Center for Nonlinear Phenomena and Complex Systems,  
Université Libre de Bruxelles, Campus Plaine - CP 231,  
Boulevard du Triomphe, Bruxelles 1050, Belgique*

**Abstract.** A new paradigm is discussed in which the discrete facet of physical phenomenology is related to the existence of localized wave solutions (*solitons*) of a underlying *metacontinuum* (unified field, physical vacuum, etc.) while the action at a distance is a manifestation of the internal stresses in *metacontinuum*. The latter is considered as thin elastic 3D layer (a shell) in the 4D geometrical space. Maxwell's equations are recovered as corollaries from the equations of the motion in the middle surface. The equation for the amplitude of transverse flexural deformation appears to be a fourth order dispersive equation similar to the one derived by Schrödinger. The localized flexural (curvature) waves propagating *over* the surface of *metacontinuum* are interpreted as particles. A possible unification of the gravitation, electromagnetic phenomena and wave mechanics is demonstrated.

### Introduction

The apparent duality between the point particles and field(s) underlies the foundation of modern physics. The Aristotelian tradition was connected with the notion of continuum but with the advance of the experimental physics the attention gradually shifted to the concept of point particles (corpuscles) moving freely in a void geometrical space. This new concept was heralded by Newton in his first two laws and it gave rise of a fruitful quantitative description of the motion of bodies known nowadays as *Newtonian Mechanics*. Yet it was Newton himself who admitted also an action at a distance when formulating the law of gravitation. Thus from the very beginning was planted the seed of dualism.

It is hard to imagine an action at a distance without some kind of a carrier. The nineteenth-century tradition was always connected with some mechanical construct. In Cauchy's and Hamilton's vision it was a lattice whose continuum ap-

---

<sup>1</sup>On leave from National Institute of Meteorology and Hydrology, Bulgarian Academy of Sciences, Sofia, Bulgaria

proximation yielded the elastic-body model.<sup>2</sup> Paying tribute to the metaphysical tradition, all of the models for the presumably absolute continuous media underlying the physical world were called “aether,” although some of them (the different elastic models, for instance) were quite different from an “aetherial fluid” (where the coinage comes from). In Maxwell’s imagination it was a medium with internal degrees of freedom. McCullagh and Sommerfeld quantified this idea as an elastic body with special rotational elasticity (see [1] for exhaustive review of these theories).

The downfall of the concept of aether began with the unsatisfactory results of Cauchy’s “volatile aether” (elastic body with vanishing dilational modulus). After Lord Kelvin came up with the model of fluid aether and its vortex theory of matter, the coinage “aether” assumed almost exclusively fluid meaning. Then the question of entrainment of aether (aether-drift) was posed and the nil result of Michelson and Morley experiment blew down the whole edifice of aether theories. However, the notion of a material carrier of the long-distance interactions could not be dismissed so easily and the conceptual vacuum was filled by the concept of what is called nowadays “physical vacuum” which possessed all the properties of the disgraced aether (e.g., action at a distance and giving birth to particles), but was deliberately exempted from the obligation to be checked for aether-drift effect. It was then advertised as a thing in itself not connected to any “primitive mechanistism.” The same conceptual load carries the coinage “field.”

On a new level, the yearning for continuum description resurrected in the Hamiltonian-Schrödinger wave mechanics. From the far side de Broglie quantified the wave properties of the particles. At that moment it was not clear how the two facets of the same object—the material particle—can peacefully coexist and it led to the concept that some kind of “pilot” wave was associated with a “point particle” representing the probability to find the latter at certain spatial position. Thus the dichotomy was deeply implanted in the physical thought and is still the dominant general attitude in the modern physics. As a result there has appeared a rather intricate web of coinages (*pseudo*, *quasi*, etc.) when certain phenomenological fact pertaining to the one facet was being explained in the terminology of the dual facet.

In our point of view, the *field* (*physical vacuum*, *aether*) can only be understood from the point of view of a material continuum where the *internal stresses are the transmitter of the long-range interactions*. In order to distinguish it from the mechanical continuous media (bodies, liquids, gases, etc.), we call the continuum-mechanics model of the unified field *metacontinuum* in the sense that it is beyond (*meta*) the observable phenomena and is their progenitor. Returning to the concept of absolute continuum will be senseless without some new paradigm concerning the “point matter.” It is not possible to imagine point particles pushing their way through infinitely stiff, virtually incompressible elastic body. Even if they could, the disturbances would be the predominant effect which has not been observed in any experiment. So the new paradigm has to deal with inventing the concept of a particle

---

<sup>2</sup>It is curious to mention here that Cauchy came up with the elasticity theory because of his attempts to constitute the aether.

which does not disturb the underlying continuum during its motion. The new concept we implant on the old tree of the absolute medium is that a particle is not something foreign to the field and immersed in the latter. Rather it is a localized wave of the field which *propagates over* the field (as a phase pattern) and is not moving *through* the field. Then we have to find phase patterns that are permanent enough in order to qualify for particles. Such individualized waves are well known from the soliton theory.

J. Scott Russell who was a contemporary of the golden age of the aether theories provided [2,3] the experimental evidence that a *nonlinear* field (2D surface of ideal fluid) can exhibit patterns of type of permanent waves with individualized compartment. The theoretical explanation was found by Boussinesq [4] and Lord Rayleigh [5] and it appeared that the balance between nonlinearity and dispersion is which sustains the long-living phase pattern. Zabusky and Kruskal [6] showed numerically that the solitary waves of KdV equation (close relative to Boussinesq equation) preserve their shapes and energies upon collisions and called them *solitons* to stress the particle-like behaviour (“quasi-particles”).

Thus the stage for the unification was set up long time ago though it was not recognized as such. The only remaining question is which is the field (*metacontinuum*) whose solitary-wave phase patterns are the particles. A valid candidate for the luminiferous field is the elastic medium because it gives a good quantitative prediction for the shear-wave phenomena (light). As it is shown in [7] and briefly outlined here, the Maxwell equations are corollary from the linearized governing equations of the metacontinuum provided that the electric and magnetic field are properly understood as manifestation of the “meta” internal stresses. The main difference from Cauchy’s elastic aether is that we consider the opposite limiting case: an elastic continuum with infinitely large dilational modulus (virtually incompressible elastic medium).

There are other candidates for the field, e.g., the *sine*-Gordon equation considered in [8,9]. It is believed to be the meson field possessing solutions of type of “quasi-particles.” Oddly enough, those localized waves are of type of fronts (kinks) which though mathematically well localized hardly fit into the common-sense picture of a particle. Yet the spatial derivatives of the solution did have the expected shape and the kinks qualify for *solitons* because the energy is conserved during the collisions. Nowadays there are many other “fields” and new are being introduced *ad hoc* for explaining one or another of the hundreds of elementary particles already known.

Here we stay firmly on the position that there is only one *metacontinuum* and the different fields are different manifestation of its internal stresses and strains. The “organization” of the matter around some terminology like “particles,” “charges,” “spins” is simply our way to simplify and thus to comprehend the complexity and richness of the interactions of localized phase patterns (solitons). For this reason we prefer to call the present attempt for an unified field theory “Soliton Paradigm.” And the most ambitious goal is to derive the electromagnetic phenomena, the gravitation and wave mechanics from a single mechanical model of a metacontinuum.

Clearly, if one is to unify the wave function with the electromagnetic phenomena one has to consider more than three spatial dimensions. In addition, the thickness

in the fourth dimension of the “material world” must be so small that it cannot be perceived by our senses. This idea was put forward by Hinton [10] but on qualitative basis only. Kalutza [11] and Klein [12] also considered additional “underdeveloped” dimensions, but they added a vector field to the Einstein equation while we add a *scalar* field to the Maxwell equations, the latter appearing here in the guise of a vector equation for shear waves in elastic continuum.

Carrying on with this idea we ask the question: “What kind of manifestation is to be expected from the fact that the material world is a thin 3D layer in the 4D geometrical space?” Naturally, the presence of “underdeveloped” dimension(s) should result into some new observables in addition to those that were sufficient to make a self-consistent picture in the Maxwellian framework. Depending on the topology of the *metacontinuum* there may be (or may not be) a number of these manifestations, connected with different spins in multi-dimensional space. The existence of at least one additional variable is inevitable, namely the amplitude of deflection of the 3D layer in direction of the fourth dimension. In Section 2 we derive an equation (“master equation of wave mechanics”) governing the said amplitude for geometrically nonlinear very thin (virtually  $N^D$ ) elastic layers in  $N + 1$  dimensions. We call this kind of mechanical construct *gossamer*. Naturally, this equation is not a follow-up from the governing equations in  $N$  dimensions, just like in the real world the Schrödinger equation is not a corollary form the Maxwell equations.

The rest of the paper is devoted to arranging a cosmological picture of the solitary waves—phase patterns. In many instances they appear to be quite similar to what the long-longed unified theory is expected to bring.

## 1. Constituting a Metacontinuum

For small velocities the Lagrangian and Eulerian descriptions of a continuum coincide and for the displacements  $\mathbf{u}$  of a Hookean elastic medium one has the linear vector wave equation

$$\mu_0 \frac{\partial \mathbf{A}}{\partial t} \equiv \mu_0 \frac{\partial^2 \mathbf{u}}{\partial t^2} = \eta \Delta \mathbf{u} + (\lambda + \eta) \nabla (\nabla \cdot \mathbf{u}) \equiv -\eta \nabla \times \nabla \times \mathbf{u} + (\lambda + 2\eta) \nabla (\nabla \cdot \mathbf{u}), \quad (1.1)$$

where  $\mathbf{u}$ ,  $\mathbf{A}$  are the displacement and velocity vectors;  $\eta, \lambda$  are Lamé’s elasticity coefficients and  $\mu_0$  is the density in material (Lagrangian) coordinates. Note that we concern ourselves for the time being only with a metacontinuum of constant elastic coefficients  $\eta, \lambda$  and density  $\mu_0$ .

The full set of physical motions governed by (1.1) includes shear and compression/dilation as well. The former are controlled by the shear Lamé coefficient  $\eta$ , while the latter—by the dilational (second) Lamé coefficient  $\lambda$ , and more specifically by the sum  $(\lambda + 2\eta)$ . The phase speeds of propagation of the respective small disturbances are

$$c = \left( \frac{\eta}{\mu_0} \right)^{\frac{1}{2}}, \quad c_s = \left( \frac{2\eta + \lambda}{\mu_0} \right)^{\frac{1}{2}}, \quad \delta = \frac{\eta}{(2\eta + \lambda)}. \quad (1.2)$$

Here  $c$  and  $c_s$  are the speeds of shear waves (*light*) and compression waves (*sound*), respectively. In the case of very large dilational modulus, the speed of sound is much greater than the speed of light and  $\delta \ll 1$ . This is the opposite limiting case than the “volatile aether” of Cauchy with  $\eta \gg \lambda$ . Then Eq. (1.1) is recast as follows

$$\delta \left( c^{-2} \frac{\partial^2 \mathbf{u}}{\partial t^2} + \nabla \times \nabla \times \mathbf{u} \right) = \nabla(\nabla \cdot \mathbf{u}), \quad (1.3)$$

and displacement  $\mathbf{u}$  can be developed into power asymptotic series with respect to  $\delta$

$$\mathbf{u} = \mathbf{u}_0 + \delta \mathbf{u}_1 + \cdots . \quad (1.4)$$

Introducing (1.4) into (1.3) and combining the terms with like powers we obtain for the first two terms

$$\nabla(\nabla \cdot \mathbf{u}_0) = 0, \quad (1.5)$$

$$c^{-2} \frac{\partial^2 \mathbf{u}_0}{\partial t^2} + \nabla \times \nabla \times \mathbf{u}_0 = \nabla(\nabla \cdot \mathbf{u}_1). \quad (1.6)$$

From (1.5) one can deduce

$$\nabla \cdot \mathbf{u}_0 = \text{const}, \quad \text{or} \quad \nabla \cdot \mathbf{A}_0 = 0, \quad (1.7)$$

which is also a linear approximation to incompressibility condition for a continuum. In the general model of nonlinear elasticity with finite deformations the incompressibility condition is imposed on the Jacobian of transformation from material to geometrical variables, but in the first-order approximation in  $\delta$  the Eq. (1.7) holds true.

From here on we omit the index ‘0’ without fear of confusion. We denote formally the term  $(\lambda + 2\eta)\nabla \cdot \mathbf{u}_1$  by  $(-\varphi)$  and recast (1.6) as dimensional form of linearized Cauchy balance, namely

$$\mu_0 \frac{\partial \mathbf{A}}{\partial t} = -\nabla \varphi + \nabla \cdot \boldsymbol{\tau}, \quad (1.8)$$

where  $\boldsymbol{\tau}$  is the deviator stress tensor for which the following relation is obtained from the constitutive relation (the Hooke law) for elastic body, namely

$$\boldsymbol{\tau} = \eta(\nabla \mathbf{u} + \nabla \mathbf{u}^T) - 2\eta(\nabla \cdot \mathbf{u})\mathbf{I}, \quad (1.9)$$

where  $\mathbf{I}$  stands for the unit tensor. For the divergence of  $\boldsymbol{\tau}$  one has

$$\nabla \cdot \boldsymbol{\tau} = -\eta \nabla \times (\nabla \times \mathbf{u}). \quad (1.10)$$

What is essential for the unification is that the linearized equations of elastic continuum admit what we call Maxwell form. The derivations here are not to be confused with McCullagh’s model of pseudo-elastic continuum (see [1,13] for references and further developments) with restoring couples by means of which he tried

to explain the unusual shape of Maxwell's equations apparently not fitting into the picture of continuum mechanics. Let us introduce the vector field

$$\mathbf{E} \stackrel{def}{=} -\nabla \cdot \boldsymbol{\tau} \equiv \eta \nabla \times (\nabla \times \mathbf{u}). \quad (1.11)$$

to which the action of the purely shear part of internal stresses is reduced. It has the meaning of a point-wise distributed body force and we shall call it "electric force." In terms of  $\mathbf{E}$ , the linearized system (1.8) yields

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \varphi, \quad (1.12)$$

involving the well known vector and scalar potentials  $\mathbf{A}$  and  $\varphi$ . In the framework of the present approach, however, these potentials are not non-physical quantities introduced merely for convenience, but rather they appear to be the most natural variables: velocity and pressure of metacontinuum. Taking the *curl* of (1.12) one obtains

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (1.13)$$

which is nothing else but the first of Maxwell's equations (the Faraday law) provided that a "magnetic induction"  $\mathbf{B}$  is defined as

$$\mathbf{B} = \mu_0 \nabla \times \mathbf{A} = \mu_0 \mathbf{H} \quad \mathbf{H} \stackrel{def}{=} \nabla \times \mathbf{A}, \quad (1.14)$$

where  $\mathbf{H}$  is called "magnetic field." From Eq. (1.11) one obtains

$$\frac{1}{\eta} \frac{\partial \mathbf{E}}{\partial t} = \nabla \times (\nabla \times \frac{\partial \mathbf{u}}{\partial t}) \equiv \nabla \times \mathbf{H}. \quad (1.15)$$

The last equation is precisely the "second Maxwell equation" provided that the shear elastic modulus of metacontinuum is interpreted as the inverse of electric permittivity  $\eta = \varepsilon_0^{-1}$ . This equation was postulated by Maxwell [14] as an improvement over Ampere's law incorporating the so-called displacement current  $\partial \mathbf{E} / \partial t$  in the Biot–Savart form. For the case of a void space, however, when no charges or currents are present, the second Maxwell equation lives a life of its own and Ampere's law plays merely heuristic role for its derivation. It is broadly accepted now that the second Maxwell equation is verified by a number of experiments. Here we have shown that it is also a corollary of the elastic rheology of the metacontinuum and is responsible for the propagation of the shear stresses (action at a distance) in *metacontinuum*.

The two main equations of evolution of Maxwell's form have already been derived. The condition  $\text{div } \mathbf{H} = 0$  (the third Maxwell equation) follows directly from the very definition of magnetic field. Similarly, taking divergence of Eq. (1.11), one immediately obtains the fourth Maxwell equation  $\text{div } \mathbf{E} = 0$ . Thus we have shown

that Maxwell's equations follow from the linearized governing equations of a Hookean elastic medium whose dilational modulus is much larger than the shear one.

## 2. A Curved Metacontinuum? The Gossamer

From the point of view of material continuum a curved medium can be considered only as being embedded into a geometrical space of at least one spatial dimension larger, e.g., the Maxwell luminiferous field can be considered as 3D material hypersurface in 4D geometrical space. It is hard to imagine a material surface which has no thickness. A 3D hypersurface is a mathematical abstraction for the material construct known as thin shell (or a membrane) when the so-called middle surface of the latter is considered. If the shell is flat then the electromagnetism will be the only phenomena to be observed in three dimensions. Let us examine now the consequences of the curvature and transverse (flexural) deflections of *metacontinuum*. The existence of another spatial dimension cannot be detected in 3D shear-waves (electromagnetic) experiments.

Following the previous sections we consider an elastic shell of a 4D material whose shear Lamé coefficient is much smaller than the dilational one (their ratio given by the small parameter  $\delta$ ). As usual in shell/plate/membrane theories the small parameter  $\varepsilon = h/L$  is the most important one, where  $h$  is the thickness of the shell and  $L$  is the length-scale of the deformations of the middle surface. The thin-layer simplification apply when  $\varepsilon \ll 1$ . The problem is to find a correct way to reduce the 4D continuum mechanics to an effective 3D mechanics for the middle surface of the shell.

In the technological applications an additional assumption is tacitly made, namely that  $L$  is large (“shallow shells”) and *shell* is called a “reasonably” thin elastic structure whose flexural deformation (deflections) are of unit order, the strains (gradients) are  $O(L^{-1})$  small and the curvatures are of second-order in smallness. Here we relieve the limitation of large  $L$  and treat the case  $1 \gg L \gg h$ . Then the deflections must be small, the strains (gradients)—of unit-order, and curvatures—large. The standard shell theory is not sufficient for describing such an object which is geometrically strongly nonlinear. As shown in [7], when deriving the shell equations for this limiting case one has to acknowledge more terms responsible for the geometric nonlinearity. At the same time the material nonlinearity is not so important because of the vanishing thickness  $h$ . In order to distinguish them from the classical shallow shells we call this kind of very thin elastic layers undergoing very high strains *gossamers*.

### 2.1. Manifestation of Underdeveloped Dimensions

We summarize here the relevant items of derivation of *gossamer's* theory. Wherever possible we keep the derivation general enough speaking about  $N^D$  layer in  $(N+1)^D$  space, but for the purposes of the present work  $N = 3$  and  $N+1 = 4$ . The Cauchy form for  $(N+1)^D$  continuum reads

$$[\rho_* a^j - P^{ij} |_{|i}] \mathbf{g}_j = 0, \quad i, j = 1, \dots, N, \quad (2.1)$$

where  $\rho_*$  is material density of the  $(N+1)^D$  continuous media filling the  $N^D$  shell. One can call it “meta-density” in order to distinguish it from the 3D density already identified as  $\mu_0$ . It is important also to note that neither  $\rho_*$  nor  $\mu_0$  have anything to do with the density of matter (number density of solitons). Here  $\mathbf{g}_j$  are the orthonormal basis of the curvilinear coordinate system;  $P^{ij}$  are the components of stress tensor;  $a^j$  are the components of the acceleration vector in the  $(N+1)^D$  space;  $\|_i$  stands for the covariant derivative in  $(N+1)^D$  space. We do not consider here  $(N+1)^D$  body forces.

Upon substituting the expressions for  $\|_i$  in terms of  $N^D$  covariant derivatives  $|_\alpha$  (see [15,16] and the extensions in [7]), the Cauchy balance law (2.1) is recast into a system for the laminar components and a scalar equation for the  $N+1$ -st component. After averaging (integrating) within the surfaces of the shell one gets to the second order  $O(\varepsilon^2)$  of approximation.

$$\rho_*\varphi^\alpha - \nabla_\beta \sigma^{\alpha\beta} = -2b_\beta^\alpha q^\beta - b_\beta^\beta q^\alpha, \quad (2.2)$$

$$\rho_*\varphi^{N+1} - \nabla_\beta q^\beta = b_{\beta\nu}\sigma^{\beta\nu} - c_{\beta\nu}m^{\beta\nu} + \rho_*\mathcal{F}, \quad (2.3)$$

where

$$q^\alpha = \frac{1}{h} \int P^{N+1,\alpha} ds, \quad \sigma^{\alpha\beta} = \frac{1}{h} \int P^{\alpha\beta} ds, \quad m^{\alpha\beta} = \frac{1}{h} \int s P^{\alpha\beta} ds,$$

$$\varphi^\alpha = \frac{1}{h} \int a^\alpha ds, \quad \psi^\alpha = \frac{1}{h} \int s a^\alpha ds, \quad \rho_*\mathcal{F} = \frac{1}{h} (P_{\text{up}}^{N+1,N+1} - P_{\text{lo}}^{N+1,N+1}).$$

Here the subscripts “up” and “lo” refer to the upper and lower shell surfaces  $s = h/2$  and  $s = -h/2$ , respectively. It is taken into account that no tractions are exerted upon the shell surfaces from the two adjacent  $(N+1)^D$  spaces. The integrals are understood as definite integrals in  $s$  between the shell surfaces while  $b$  and  $c$  are functions of the “surface” coordinates only. The notation  $\nabla$  stands for a  $N^D$  covariant derivative in which the coefficients that may possibly depend on the transverse coordinate are already averaged.

The system (2.2), (2.3) is coupled by the “momentum-of-impulses” which can be derived from (2.1) to the same asymptotic order  $O(\varepsilon^2)$  after multiplying it by  $s$  and integrating across the shell, namely

$$-\rho_*\psi^\alpha + \nabla_\beta m^{\alpha\beta} = q^\alpha, \quad (2.4)$$

which allows us to exclude the quantity  $q^\alpha$  from the governing system and to obtain

$$\rho_*\varphi^\alpha = \nabla_\beta \sigma^{\alpha\beta} - 2b_\beta^\alpha \nabla_\nu m^{\beta\nu} - b_\beta^\beta \nabla_\nu m^{\alpha\nu}, \quad (2.5)$$

$$\rho_*\varphi^{N+1} = \nabla_\beta \nabla_\nu m^{\beta\nu} + b_{\beta\nu} \sigma^{\beta\nu} - c_{\beta\nu} m^{\beta\nu} + \rho_*\mathcal{F} - \rho_* \nabla_\alpha \psi^\alpha. \quad (2.6)$$

Here the notion of the geometrization of physics becomes transparent. If the observer is confined to the  $N^D$  space of the middle surface he will appreciate the presence of the  $N+1$ -st dimension as additional terms in balance law (2.5), (2.6)

which terms are not present in the Cauchy form for the  $N^D$  continuous media.<sup>3</sup> The said terms are proportional to the different curvature forms and this is the quantitative expression of Riemann–Clifford [17,18] idea that the physical laws are manifestation of deformations of the geometrical space.

## 2.2. Elastic Shell with Momentum Stresses

According to the Kirchhoff-Love hypothesis, the displacements  $u_\alpha$  in the shell space are related to the  $N^D$  displacements  $\tilde{u}_\alpha$  in the shell middle surface as follows

$$u_\alpha = \tilde{u}_\alpha - s\nabla_\alpha\zeta, \quad u_{N+1} = \zeta, \quad (2.7)$$

where  $\zeta$  stands for the shape function of deformation (deflection) of the middle surface in direction of  $(N + 1)$ -st dimension. This hypothesis is pertinent to the overall  $o(\varepsilon)$  approximation since it amounts to neglecting terms proportional to  $s^2$ . Then we obtain

$$\varphi_\mu = \frac{\partial^2 \tilde{u}_\mu}{\partial t^2}, \quad \nabla^\mu \psi_\mu = -\frac{h^2}{12} \frac{\partial^2 \Delta \zeta}{\partial t^2}, \quad \varphi_{N+1} = \frac{\partial^2 \zeta}{\partial t^2}. \quad (2.8)$$

The rotational inertia  $\nabla^\mu \psi_\mu$  is of second order which justifies neglecting it in comparison with the transverse inertia  $\varphi_{N+1}$ .

In terms of coordinates that are measured along the arcs of the middle surface (precisely the material Lagrangian coordinates), the second fundamental form assumes the following simple form

$$b_{\alpha\beta} = \nabla_\alpha \nabla_\beta \zeta. \quad (2.9)$$

Note that for coordinates not coinciding with the arcs, the expression of the second fundamental form involves nonlinear terms.

It is time now to couple the Cauchy equations with constitutive relations. Unlike the Cauchy form, the full nonlinear constitutive relations cannot be derived in Eulerian framework. It goes beyond the framework of the present work to derive them in full detail, especially as far as the material nonlinearity for the laminar components is concerned. We resort here to *linear* constitutive relations in the form (see [19])

$$\sigma^{\alpha\beta} = (\lambda_* + \eta_*)g^{\alpha\beta}(\nabla_\nu \tilde{u}^\nu) + \eta_* \nabla^\beta \tilde{u}^\alpha, \quad (2.10)$$

$$m^{\alpha\beta} = -D_* \nabla^\alpha \nabla^\beta \zeta, \quad D_* = \frac{\eta_* h^2}{12}. \quad (2.11)$$

where  $D_*$  is called stiffness of shell. Here asterisks designate the  $(N + 1)^D$  material properties. If the middle surface of *gossamer* is to behave as the Maxwell luminiferous field then the following relations must hold true

$$\frac{\lambda_*}{\rho_*} = \frac{\lambda}{\mu_0}, \quad \frac{\eta_*}{\rho_*} = \frac{\eta}{\mu_0}, \quad \frac{D_*}{\rho_*} = \frac{D}{\mu_0}.$$

---

<sup>3</sup>If one was lucky enough to have already the tensor analysis developed and the notion of continuous medium (with its Cauchy balance) already established.

Upon introducing (2.10), (2.11) into Cauchy equations (2.5), (2.6) we get for the laminar components

$$\begin{aligned} \mu_0 \frac{\partial^2 \tilde{u}^\beta}{\partial t^2} &= (\lambda + \eta) \nabla^\beta (\nabla_\nu \tilde{u}^\nu) + 2\eta \Delta \tilde{u}^\beta \\ &- D [2\nabla^\beta (\nabla \zeta \cdot \nabla (\Delta \zeta)) + (\Delta \zeta) \nabla^\beta \Delta \zeta], \end{aligned} \quad (2.12)$$

and for the amplitude  $\zeta$  of normal deflection in direction of  $N + 1$ -st dimension

$$\mu_0 \frac{\partial^2 \zeta}{\partial t^2} = \mu_0 \mathcal{F} + D \left[ -\Delta \Delta \zeta + (\nabla_\beta \nabla_\delta \zeta) (\nabla^\beta \nabla_\mu \zeta) (\nabla^\mu \nabla^\delta \zeta) \right] + \frac{\mu_0}{\rho_*} \sigma^{\beta\alpha} (\nabla_\beta \nabla_\alpha \zeta). \quad (2.13)$$

Here we shall not consider the case of nontrivial tractions on the shell surfaces which requires additional assumptions about their asymptotic order. We keep, however, the term responsible for the normal pressure (parameter  $F$ ).

If we develop with respect to the powers of the small parameter  $\delta$  in the same manner as in the previous section, we get from (2.12) and (2.10) that

$$\nabla^\beta \nabla_\nu u^\nu = 0 + O(\delta) \implies \nabla_\nu u^\nu = \kappa_0 = \text{const} \implies \frac{\mu_0}{\rho_*} \sigma^{\alpha\beta} = \sigma_0 g^{\alpha\beta}, \quad \sigma_0 = \eta \kappa_0,$$

where  $\kappa_0$  is dimensionless divergence of the displacement field in the middle surface. Note that  $k_0 > 0$  means uniform dilation of the middle surface, while  $k_0 < 0$  reflects the case of uniform compression. Thus, in the lowest asymptotic order of  $\delta$  we are faced with uniform compression/dilation in the middle surface of *gossamer* and with constant membrane stress  $\sigma$  acting in the middle surface. This allows to effectively decouple the laminar deformations  $\tilde{u}^\alpha$  from the deflection  $\zeta$  even without requiring special relation between the scales for  $\zeta$  and  $\tilde{u}$ . In its turn the term in (2.13) containing  $\sigma^{\alpha\beta}$  becomes simply  $\sigma_0 \Delta \zeta$ . Because of the large dilational modulus, the laminar deformations appear to be orthogonal to the transverse ones to the first asymptotic order.

From here on the “tildes” denoting the laminar variables will be omitted without fear of confusion.

### 2.3. The Dispersive Equation of Wave Mechanics

The only term which looks unusual in (2.13) is the cubic nonlinear term. However, in dimension one the equation under consideration is exactly the cubic nonlinear Boussinesq equation. In order to benefit from the vast knowledge accumulated for equations of type of Boussinesq, we replace in a paradigmatic fashion the cubic term in (2.13) by  $(\Delta \zeta)^3$ . It is beyond doubt that the qualitative behaviour of the solutions will be quite similar. However, the extent to which they will be quantitatively close as well, remains to be verified. Then we arrive at

$$\mu_0 \frac{\partial^2 \zeta}{\partial t^2} = \mu_0 \mathcal{F} + D \left[ -\Delta \Delta \zeta + (\Delta \zeta)^3 \right] + \sigma_0 \Delta \zeta. \quad (2.14)$$

We render the last equation dimensionless by introducing the scales

$$\zeta = L\zeta', \quad \mathbf{x} = L\mathbf{x}', \quad t = \frac{L}{c_f}t', \quad c_f = \sqrt{\frac{\sigma_0}{\mu_0}} \equiv c\sqrt{|k_0|}, \quad (2.15)$$

where  $c_f$  has dimension of velocity. Note that the scale for  $\zeta$  and the length scale of the localized wave coincide (the length  $L$ ), if one looks for commensurable effects (balance) of the nonlinearity and dispersion. This fits perfectly the original assumptions for *gossamer*: small deflections of order  $L$ , unit strains and large curvatures of order  $L^{-1}$ . Finally, the dimensionless form of the wave equation of *gossamer* reads

$$\frac{\partial^2 \zeta'}{\partial t'^2} = \mathcal{F}' + \beta \left[ -\Delta \Delta \zeta' + (\Delta \zeta')^3 \right] + \text{sign}[k_0] \Delta \zeta', \quad (2.16)$$

where  $\beta = D|\sigma_0|^{-1}L^{-2}$  is the dispersion parameter and  $\mathcal{F}' = \mathcal{F}L\mu_0/|\sigma_0|$  is the dimensionless value of the normal load (hydrostatic pressure). From here on the primes denoting dimensionless variables are henceforth omitted without fear of confusion. Eq. (2.16) is our “master equation of wave mechanics.”

Now  $\beta$  is the only intrinsic non-dimensional parameter and if it is significant, then without losing the generality it may be set equal to one. This defines the length scale  $L$  of the particle-waves as

$$L \sim \sqrt{\frac{D}{\sigma_0}} \sim \frac{h}{\sqrt{|k_0|}} \implies \varepsilon = \frac{h}{L} \simeq \sqrt{|k_0|}.$$

The last relation shows that the model is applicable only if the longitudinal compression of the *gossamer* is of order of the main small parameter, namely  $\varepsilon \sim \sqrt{|k_0|} \ll 1$ .

Some remarks are due concerning the “master wave equation” (2.16). Its linear part has the form originally proposed by Schrödinger (Eq. (4) from [20]). Naturally, Schrödinger himself mentioned the analogy with the plate equation and injected a remark in the cited paper. It was later on when the interpretation of the wave function as probability distribution has been introduced and the now standard form of the equation involving a complex wave function was introduced. In order to distinguish the originally derived equation from the “canonical” form we call the linear wave equation containing fourth-order dispersion Schrödinger’s Schrödinger Equation (SSE). The derivation of Schrödinger was rather heuristic. Here we have arrived at qualitatively similar equation but bearing in mind a “palpable mechanical construct.” The wave function now has a simple meaning: the deflection of *gossamer*’s middle surface in direction normal to it (alongside of the  $(N + 1)$ -st dimension). These are *par excellence* curvature waves envisaged by Riemann [17] and Clifford [18].

### 3. The Cosmos of Localized Structures in Gossamer (Dynamics of Patterns in the Metacontinuum)

#### 3.1. A Model for Loading the Metacontinuum

A way to achieve a homogeneous and isotropic loading of *gossamer* is to con-

sider a large  $N^{\mathcal{D}}$  bubble (hypersphere) subjected to hydrostatic pressure from the adjacent  $(N + 1)^{\mathcal{D}}$  spaces. We call this hypersphere *Universe*. We assume that the bubble is compressed from the outside  $(N + 1)^{\mathcal{D}}$  (negative membrane stress in the middle surface). A discussion on the case when the model of *Universe* is an inflated bubble can be found in [7]. It is however, a kind of artificial since to get there the familiar *sech*-solitons one has also to change the sign of the cubic term. Hence the “inflated-bubble” model appears to have only heuristic significance. The motionless (equilibrium) state is characterized by the balance between the membrane tension and the hydrostatic compression creating a negative membrane tension in the middle surface, namely

$$\sigma_0 \frac{N}{R} = -\rho_* |\mathcal{F}| \quad \implies \quad k_0 = -\frac{|\mathcal{F}|R}{Nc^2} < 0 .$$

Here  $R$  is the radius of the bubble made dimensionless by the scale  $L$ , and  $N$  is the dimension of the middle surface of the *gossamer* ( $N = 3$  for the physical Universe).

It is clear that the dimensionless force  $\mathcal{F}$  must be small enough (of order of the inverse of the dimensionless radius of Universe). We introduce the relative displacement  $\bar{\zeta} = \zeta - R$ . Since we are interested in an Universe whose scale is much larger than the size  $L$  of its particles then the dimensionless radius  $R$  is extremely large. Hence its derivatives are very small and can be neglected in the cubic term. For all purposes the shell can be treated as plane. Then Eq. (2.13) is reduced to the following

$$\frac{\partial^2 \bar{\zeta}}{\partial t^2} = -\Delta \bar{\zeta} + \beta [(\Delta \bar{\zeta})^3 - \Delta \Delta \bar{\zeta}] , \quad (3.1)$$

and  $\beta = 1$  without loss of generality.

### 3.2. Flexural Localized Structures—The Flexons

In the stationary case the localized waves under question have spherical symmetry and for them the following dimensionless boundary value problem in infinite domain is posed

$$b - b^3 + \frac{1}{r^{N-1}} \frac{d}{dr} r^{N-1} \frac{d}{dr} b = 0, \quad b \rightarrow 0 \quad \text{for} \quad r \equiv |\mathbf{x}| \rightarrow \pm\infty , \quad (3.2)$$

where  $b = \Delta \zeta$  is the curvature of the nontrivial transverse elevations/depressions. The linearized version of Eq. (3.2) possesses along with the trivial solution a localized non-trivial one (the *sinc* function):

$$\zeta = ar^{-1} \sin r , \quad (3.3)$$

where  $a$  is an arbitrary constant. This means that a linear bifurcation takes place which makes our problem different from the classical soliton problems where a hard (nonlinear) bifurcation is at hand. The *sinc*-shape solution (3.3) has been just recently interpreted as a “single event” of the Schrödinger equation [21]. In fact the present work justifies using the dispersive master equation (or Schrödinger equation) not as an equation for the probability density of a particle, but rather as a field equation (in

the same fashion as *sine*-Gordon equation is used after [8]). It has to be pointed out that in our model we do not impose the shape of the potential as it is usually done in quantum mechanics.

We have solved (3.2) by means of an numerical algorithm based on the Method of Variational Imbedding (MVI) developed in [22] with application to homoclinics of Lorentz system (see also [23] for application to dissipative solitons). Because of the slow algebraic decay of the tails of the solution the interval has to be truncated at very large  $r$ . At the same time the grid has to be dense enough to allow resolving the oscillations. The results presented here employ grids with up to 40000 points and spacing 0.01. The solution turned out to be very sensitive to the mesh size and the magnitude of the “actual infinity,” so some additional refinement could change the the amplitudes of solitons presented in Fig. 1 with couple of percents. Typically for nonlinear bifurcation problems, more than one nontrivial solution appear for the same values of the governing parameters. The question of the number of non-trivial solutions is of prime importance for the physical applications. Here we have found solutions with discrete set of amplitudes. This result suggests that for larger amplitudes the support of the main peak of the solution shrinks, i.e. the larger particles have shorter Compton wavelength. With the increase of the amplitude the behaviour of the solution in the origin  $r = 0$  approaches  $r^{-1}$  which is a singular solution bringing a balance between the second order operator and cubic nonlinear term in (3.2). The physical significance of such kind of singular solutions is not yet understood in the framework of the proposed here paradigm and we did not go for larger amplitudes.

We identify the obtained here flexural solitary waves of the governing equations of gossamer as the *particles*. The fact that the metacontinuum is elastic does not necessarily mean elastic interaction of the particles (see the demonstration of this fact for the Boussinesq equation in [24]). As we show in what follows, the model under consideration is a conservative one, but to prove that our flexural localized waves live up to the definition of *solitons* is necessary either to find the two-soliton solution of (3.1) or to demonstrate the interactions of *flexons* numerically. We have done neither of these two things because the analytical techniques simply do not work for negative membrane tension. At the same time the 1D reduction of our model does not possess localized solutions. Numerical solution of a 2D problem with the above mentioned requirements for the grid size in each direction would have needed enormous computational resources. Until their solitonic properties are strictly established we will call the localized waves discovered here *flexons* which carries also a hint of their origin (flexural deformations or deflections).

As it will be seen later on, the amplitude of a *flexon* is directly related to the *mass* of particle. Naturally, a *flexon* with negative amplitude will be an *anti-particle*. The simplified Eq. (3.2) does not distinguish between particles and anti-particles. However, in the original model of *Universe* a slight difference between particles and anti-particles is to be expected due to to the curvature of the undisturbed bubble. Either the particles or the anti-particles will have a better chance to appear depending on which of them minimize the stored elastic energy of the shell.

One sees in Fig. 1 that the amplitudes (and hence—the masses) of flexons can

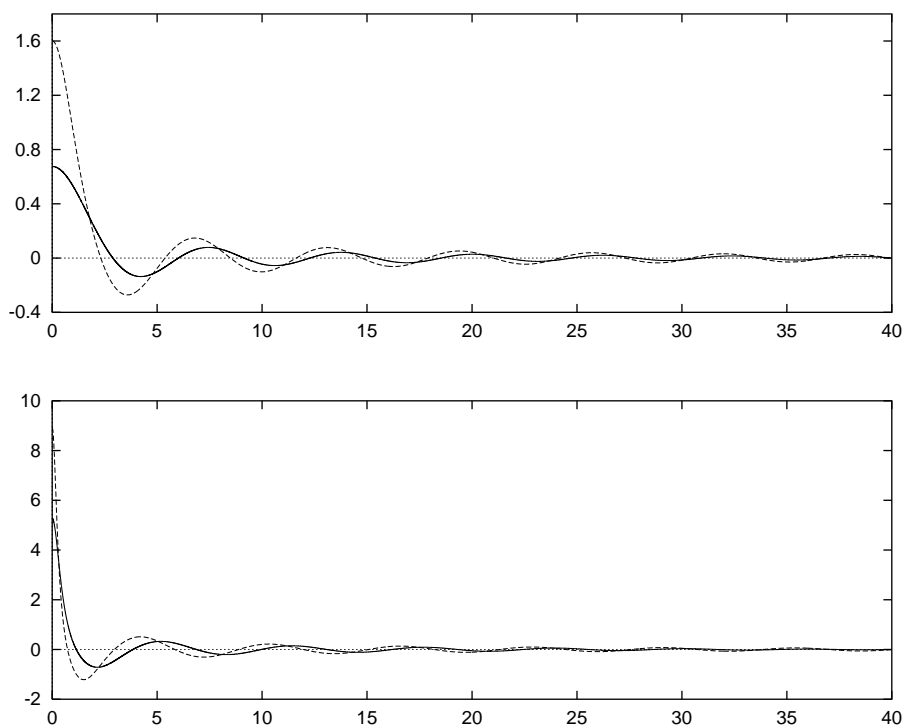


Fig. 1. Flexion solution with small amplitudes (up) and large amplitudes (down).

be quite different. To excite a bigger *flexion*, one has to invest greater energy as initial condition of the iterations. Respectively, in the iterative process some of the bigger particles decay to smaller emitting part of their energy. In this instance, the situation is qualitatively similar to relationship of nucleons and quarks.

### 3.3. Localized Shear Waves in the Middle surface: Torsion Solitons (*Twistons*)

Although the way we load the gossamer is the simplest one (a uniform hydrostatic pressure), it turns out that a complex “cosmological” picture appears with host of different localized waves. Bifurcation and symmetry breaking does not affect just the transverse deformation. Here we provide an example of what can happen in the middle surface. Unfortunately the material nonlinearity has not yet been incorporated in the model and the linearized equations for laminar components remain not closed due to the presence of the pressure-like term  $\nabla\varphi$ . For that reason the presented here solution has mainly qualitative heuristic bearing. By direct inspection one finds that the vector

$$\mathbf{u} = \left\{ \frac{x(z^2 - y^2)}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}, \frac{y(x^2 - z^2)}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}, \frac{z(y^2 - x^2)}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} \right\}$$

satisfies the “incompressibility condition”  $\nabla \cdot \mathbf{u} = 0$ . The vector lines corresponding to this solution resemble much a vortex or a bundle. This deformation field creates

an electric field given by

$$\mathbf{E} = \frac{12\mathbf{u}}{x^2 + y^2 + z^2},$$

which is singular in the origin. It decays at infinity as  $r^{-2}$  which is in accord with the electromagnetic theory.

With the vortex-like solutions topological charges can be associated but it goes beyond the scope of present work to give the details, moreover that different topological charges can be defined and this still awaits its mechanical explanation. We call the localized solutions of vortex type *twistons* in order to distinguish them from the fluid vortices. The obvious symmetry of the linearized problem shows that the charge can be positive or negative. Depending on their charges, two *twistons* repel or attract each other (just as two vortices do). The latter means that the presence of a *twiston* can only be experienced by another *twiston*. The neutral particles (*flexons*) remain unaffected by the shear deformations in the middle surface of *gossamer* to the lowest asymptotic order. However, the  $O(\delta)$  coupling between the laminar and transverse (flexural) deformations can cause a slight elevation (depression) of the *gossamer* surface in the region of localization of a *twiston*. In other words, the *twiston* has its own mass which can either be positive or negative and is much smaller than the mass of the *flexon* (the neutral particle). By analogy one can call it “mass of electron/positron.” Massive charged particles can be produced when a *twiston* “nests on” a *flexon* and then the mass is the superposition of the amplitude of the neutron (*flexon*) and the amplitude of the flexural deformation associated with the *twiston* (positron or electron).

There is no much sense going on here with the details before the material nonlinearity of *metacontinuum* is established. In addition, the singularity of vortex solutions suggests that some higher-grade elasticity is to be admitted in order to change the behaviour of the solution for  $r \rightarrow 0$ , e.g., in the following fashion

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} + \Phi(\nabla \mathbf{u}, \Delta \mathbf{u}) = \Delta \mathbf{u} - \nabla \varphi - \chi \Delta \Delta \mathbf{u}, \quad (3.4)$$

where  $\Phi$  stands for the yet unspecified nonlinearity and  $\chi$  is the dispersion coefficient.

Here a comment is due on the limiting speed of light. Clearly, the speed of light is not a limiting celerity for a *flexon*. A truly neutral *single* particle should not be affected by the speed of shear waves (light) as being orthogonal to shear deformation. However, as far as the charged particles are concerned, the speed of light is indeed the limiting speed because the *twistons* are solutions of the nonlinear wave equations for the laminar components of displacements. Hence an atom containing charged particles cannot exceed the speed of light since above it the charges (*twistons*) will be disjointed from the flexural “humps” that carry them.

### 3.4. Density Solitons

Alongside with the flexural and torsion solitary waves, one must expect also solitary waves connected with the compressibility of *metacontinuum*. Hence a quantitative numerical solution for the density solitons has not been attempted by us. How-

ever, some qualitative conclusions can be reached on the basis of the known properties of the compression waves in solids. First of all, the speed of these waves is limited by the speed of sound of the *metacontinuum* and the steepness of a density wave increases with its celerity. This means that the swifter the movement of the wave—the smaller its spatial extent. On the other hand, the density solitons are expected to interact almost insignificantly with the matter (the *flexons* and *twistons*) and in this instance they will resemble to a great extent the behaviour of the neutrino. This qualitative description allows us to consider the all pervading compression/dilation motion (orthogonal to the matter) as quite similar to what the ancient School of Stoa called *pneuma* (see, e.g., [25]).

### 3.5. The Shell Membrane Tension or The Gravitation

According to the picture drawn here, the particles are localized elevations (humps) of the gossamer surface. Due to the presence of a *flexon* of shape  $\zeta$  situated at certain geometrical position, the material points will experience attractive force proportional to  $|k_0|\nabla\zeta$ . For a single *flexon* (see Fig. 1), this force will not be monotone. The bare fact is that nobody has measured the gravitation force between two elementary particles and it is not clear whether the attractive force is monotone in the intra-atomic regions. In fact, the gravitation law is established only for bodies (ensembles of *flexons*). So one has to consider only the flexural-deformation field as averaged over the different positions of the particles-flexons. It is to be expected that the total average is positive (attraction), since the positive humps of the *flexon* amplitude are larger than the negative. Taking the *sinc*-shape for qualitative purposes one has approximately

$$\zeta = \sum_{\mathbf{x}_\alpha \in D_B} \frac{a_\alpha f|\mathbf{x}_\alpha - \mathbf{x}|}{|\mathbf{x}_\alpha - \mathbf{x}|} \approx \frac{g}{|\mathbf{x}|}, \quad g \sim \gamma \int f(|\mathbf{x}_\alpha|) d\mathbf{x}, \quad (3.5)$$

where  $D_B$  is the region occupied by the body. Here it is acknowledged that  $|\mathbf{x}| \gg |\mathbf{x}_\alpha|$  and an assumption is made that the centers of particles are randomly distributed in the region of body with number density  $\gamma$ . Note that if the *flexons* were strictly *sinc*-shaped, then the constant  $g$  would have been equal to zero for randomly dispersed atoms (integral of *sine* is zero). For a *flexon* obtained here  $g \neq 0$  because its amplitude in the origin is larger than of the *sinc*, so that after rescaling by  $|\mathbf{x}|$  the *flexon* shape gives for the integral in (3.5) a positive quantity.

Thus the force that is experienced by a material point of the *gossamer* is proportional to  $\nabla\zeta \sim G|\mathbf{x}|^{-2}$ ,  $G = |k_0|g$  and both ingredients of  $G$  have extremely small values. The exact value of the “gravitational constant”  $G$  can be specified only after the averaging procedure is performed with the appropriate rigor. The membrane force acts to “pull” the material points of the *gossamer* towards the center of the particle system under consideration. Thus we arrive at Newton’s inverse-square law of gravitation which is a manifestation of the fact that the shell is a 3D continuum.

In fact, the attraction between particles arises out of the disturbances they introduce in the uniform membrane anti-tension acting in the shell, i.e. we discover a quantitatively reversed but philosophically identical picture to the concept of Mach

that the gravitation is due to the interaction with the quiescent matter at the rim of Universe. Here apply the words of Maxwell from the end of Part IV of [14] "...that the presence of dense bodies influences the medium so as to diminish this energy whatever there is a resultant attraction." Indeed, the presence of humps over the gossamer surface influences the medium so as to diminish the stored energy and there arises a resultant attractive force.

In our model there is no place for gravitational waves because the membrane tension is negative hence the deflection waves would propagate infinitely fast were the dispersion not present. Bizarrely, the particles themselves can loosely be called "gravitational waves" since part of the forces constituting the particles is the gravitation (membrane anti-tension). In other words, a particle is a localized nonlinear wave sustained from the balance between the generating effect of the membrane anti-tension on the one hand, and the restraining effect of the cubic nonlinearity and dispersion on the other.

### 3.6. Dispersion and "Red Shift"

It is peculiar that the Boussinesq equation (3.4) possesses *localized* solutions (see [24,26]) that propagate with the characteristic speed (photons?) and undergo some aging ("red shift") in the sense that their support increases while the amplitude decreases. Far from the source, one cannot distinguish between the red-shifting due to dispersion or to a Doppler (effect if present). This means that if a dispersion is present then the "red shift" can be alternatively explained without the help of "Big-Bang" hypothesis.

### 3.7. Estimating the Constants of Metacontinuum

The density of *metacontinuum* is the magnetic permeability  $\mu_0$ . It is well known that the whole electrodynamics can be built without specifying the dimension of  $\mu_0$ . In our paradigm it is exactly to be expected that way, since  $\mu_0$  is a *meta* quantity. Its dimension is not needed anywhere in the model and the density of matter (being the number density of solitons) has nothing to do with it.

The shear Lamé coefficient  $\eta$  is the inverse of the electric permittivity  $\varepsilon^{-1}$ . The thickness of *gossamer* is proportional to Plank's constant  $h \sim \hbar c^{-1} \mu_0^{-1}$ . If there is a dispersion coefficient in the equations for laminar displacements it can be estimated from the Hubble constant. The dilational Lamé coefficient  $\lambda$  can be estimated only from an "acoustic" experiment in the metacontinuum.

## 4. Dynamics of Patterns in Metacontinuum

### 4.1. The Hamiltonian Formulation

Consider Eq. (3.1) in the domain  $D$  with boundary conditions for the wave-amplitude  $\zeta$

$$\zeta = \psi_0(\mathbf{x}), \quad \Delta\zeta = \psi_2(\mathbf{x}), \quad \text{for } \mathbf{x} \in \partial D. \quad (4.1)$$

The trivial boundary conditions correspond to an isolated system of wave-particles. A Hamiltonian representation is readily derived upon multiplying (3.1) by  $\Delta\zeta_t$  and

integrating over the infinite domain.

$$H \equiv \frac{1}{2} \int_D \left[ (\nabla \zeta_t)^2 - (\Delta \zeta)^2 + \frac{1}{2} \beta (\Delta \zeta)^4 + \beta (\nabla \Delta \zeta)^2 \right] d^N \mathbf{x}. \quad (4.2)$$

For boundary conditions that do not depend on time, the total energy  $H$  is conserved  $dH/dt = 0$ .

The energy functional (4.2) is not positive definite, but unlike the Boussinesq wave equation the solution will not blow up in finite time since the quartic nonlinearity dominates the quadratic term saturating the growth of solutions. Although the second-order term in (3.1) is of improper sign the equation is not unstable with respect to short wave lengths because of the presence of fourth spatial derivatives with the proper sign.

The wave mass and wave momentum are defined as

$$M \equiv \int_{-\infty}^{\infty} \Delta \zeta d^N \mathbf{x}, \quad \mathbf{P} \equiv \int_{-\infty}^{\infty} \zeta_t \nabla \Delta \zeta d^N \mathbf{x}. \quad (4.3)$$

The concept of *pseudomomentum* in continuum mechanics was elaborated recently [27,28] especially in connection with the interpretation of localized waves as “quasi-particles” and numerous featuring examples are presented in [29]. For this model the meaning of *mass* is as “mass of curvature” which once again is in the vein of Riemann proposal. The *mass* is conserved if  $\partial \zeta_{tt} / \partial n = 0$  at  $\partial D$  which is a natural requirement for an isolated system. For the *pseudomomentum* we have

$$\begin{aligned} \frac{d\mathbf{P}}{dt} &= \oint_{\partial D} \mathbf{n} \left[ -\frac{1}{2} (\nabla \zeta_t)^2 - \frac{1}{2} (\Delta \zeta)^2 + \frac{\beta}{4} (\Delta \zeta)^4 + \frac{\beta}{2} (\nabla \Delta \zeta)^2 \right] d\mathbf{s} \\ &\quad - \oint_{\partial D} \beta (\nabla \Delta \zeta) \frac{\partial \Delta \zeta}{\partial n} d\mathbf{s} = \mathcal{F}_{\text{psu}}, \end{aligned} \quad (4.4)$$

where  $\mathcal{F}_{\text{psu}}$  is called *pseudoforce* and  $\mathbf{n}$  is the outer normal to the region  $D$ . The *pseudomomentum* is conserved if the *pseudoforce* is equal to zero. Note that even for an isolated system the *pseudomomentum* is not conserved if the localized patterns hit the boundary and rebound.

#### 4.2. From Metadynamics of Underlying Continuum to Dynamics of Centers of Localized Structures

The main significance of the Hamiltonian formulation is that it provides the means to build up the dynamical model for the *discrete* phase objects (*solitons*). If the shape of localized wave is known, the Hamiltonian dynamics for the discrete system of centers of “particles” can be derived from (3.1) with a good approximation provided that they do not interact so strongly as to change appreciably their shapes. For two localized waves the wave amplitude can be decomposed as follows:

$$\zeta = F_1(\mathbf{x} - \mathbf{X}_1(t)) + F_2(\mathbf{x} - \mathbf{X}_2(t)) + F_{12}(\mathbf{x} - \mathbf{X}_1(t), \mathbf{x} - \mathbf{X}_2(t)). \quad (4.5)$$

Here  $F_i$  are the shape functions of the waves-particles and  $\mathbf{X}_i(t)$  are the trajectories of their centers. When the particles are far from each other  $F_{12}$  is negligible. In fact

$F_{12}$  must be considered only in the cross-section of the collision of two particles and we leave the problems connected with this case for future study.

Now the time derivative of each shape function can be expressed as follows

$$\frac{dF_i}{dt} = -\nabla F_i \cdot \frac{d\mathbf{X}_i}{dt} \quad (4.6)$$

and the discrete Hamiltonian contains quadratic forms of the velocities of centers

$$\sum_i \mathcal{A}_i \cdot \dot{\mathbf{X}} \dot{\mathbf{X}}, \quad (4.7)$$

where the matrix  $\mathcal{A}_i$  is positive definite. Then (4.7) can be interpreted as definitive relation for the kinetic energy of the center of particle. Respectively,  $F_2$  will contribute a term which depends on the relative position of particles and plays the role of potential of interaction. In absence of interactions, the Euler–Lagrange equations for discrete system of centers of *solitons* give  $\ddot{\mathbf{X}} = 0$  and a *soliton* will propagate with constant phase velocity. Thus for isolated particles the first Newton law is recovered.

#### 4.3. FitzGerald-Lorentz Contraction and Contraction of Flexons

The FitzGerald-Lorentz contraction (FG–L, for brevity) is a standard feature with the soliton solutions of generalized wave equations, e.g., SG, Boussinesq, etc. It is especially well seen for the *sine*-Gordon equation where the “quasi-particles” are called “relativistic”<sup>4</sup> meaning that their contraction is exactly proportional to the Lorentz factor. Naturally, the above discussed localized shear waves are subject precisely to the same factor of contraction. This holds true also for the length scales of interaction forces (e.g., the Coulomb force).

When a *flexon* propagates with a constant celerity  $V$  its measure in the direction of propagation must be shortened by the factor (see (2.15), (3.1))

$$\left(1 + V^2/c_f^2\right)^{-\frac{1}{2}}, \quad (4.8)$$

where  $c_f$  is the pseudo-velocity corresponding to the negative membrane tension. Respectively the amplitudes of *flexons* (related to their particle-wave masses) are increased roughly by the same factor. Since  $c_f$  is much smaller than the speed of light, the contraction of flexons must be felt for smaller phase velocities than the Lorentz contraction. On the other hand—contrary to the FG–L contraction—there is no singularity in the expression of *flexon* contraction (4.8). However, the new type of contraction is much less important for the contraction of the bodies since it only affects the scale of gravitational interaction which is anyway the weakest and is not important in defining the intra-atomic scales. Yet, it is quite possible that some small effect could be felt due to the flexural contraction. It will not be a surprise if the

---

<sup>4</sup>Another specimen from the glittering panoply of coinages which is misleading in this context since SG is a model of an absolute field

weak but persistent deviations found in [30] from the nil effect of the Michelson-Morley experiment turn out to be the result of this additional contraction.

## 5. A First-Order Experiment for Detecting the Doppler Effect

To use interferometry for verification of the Doppler effect was suggested by Maxwell [31]. It was believed that discovering a Doppler effect will prove the existence of an absolute medium at rest. The experiment was implemented first by Michelson [32] and nil effect was observed. It was later on refined by Michelson and Morley [33] (MM, for brevity) and the absence of the expected type of interference was confirmed more decisively. In our opinion the nil effect of MM experiment cannot disprove the existence of absolute medium because it seeks for a second-order effect in the small parameter  $d = v_e c^{-1} \approx 10^{-4}$  ( $v_e$  stands for the velocity of Earth with respect to the quiescent medium). This was specially pointed out by Maxwell [31] way before the experiment was performed. He proved that employing only a single ray with splitting and reflections inevitably renders the sought effect of second order  $d^2$  because the light travels along a closed path. The only conclusion that can be drawn from the nil effect is that in the medium where the light is being propagated there occurs an apparent contraction of the spatial scales in the direction of motion of the source (FitzGerald-Lorentz contraction) proportional to the factor

$$1 - \frac{v_e^2}{2c^2} \approx \sqrt{1 - \frac{v_e^2}{c^2}}$$

which exactly compensates for the expected second-order effect.

In fact MM result strongly suggests the existence of an absolute medium because the FitzGerald-Lorentz contraction is a mandatory effect in a field theory based on the nonlinear generalizations of the D'Alembert wave equation (see the comments in a preceding section). Thus the soliton paradigm provides a most natural explanation of the nil effect of MM experiment. If the “material bodies” (e.g., the arms of the interferometer) are *bound states* of solitons, then they are contracted in the direction of motion because the solitons themselves and the inter-soliton distances are. The latter are defined mainly by the electromagnetic forces whose length-scales also suffer contraction. Hence the distance that must be traveled by the light through the *quiescent* metacontinuum between two soliton formations (the emitter and receptor), is indeed shorter in the direction of motion in comparison with the path traveled in transverse direction.

Note that the *metacontinuum* itself is not contracted and that is why the speed of light has a constant value in each coordinate system connected with the propagating objects—*solitons*.

The real proof of the presence of a *metacontinuum* would be an experiment for the first-order effect. Such an experiment can only be achieved if *two different* sources of light are employed with sufficiently synchronized frequencies. There are available in the market lasers with the required stabilization (e.g., the *HeNe* Model 200 of Coherent Components Group) but it goes beyond the frame of the present

work to present the details on hardware. As the sought effect is of order of  $10^{-4}$  then if the two sources are synchronized up to  $10^{-6}$ , the accuracy would be of order of 1%. In this sense we will call such sources “identical.” Here we give in Fig. 2 the principle scheme of a possible first-order experiment.

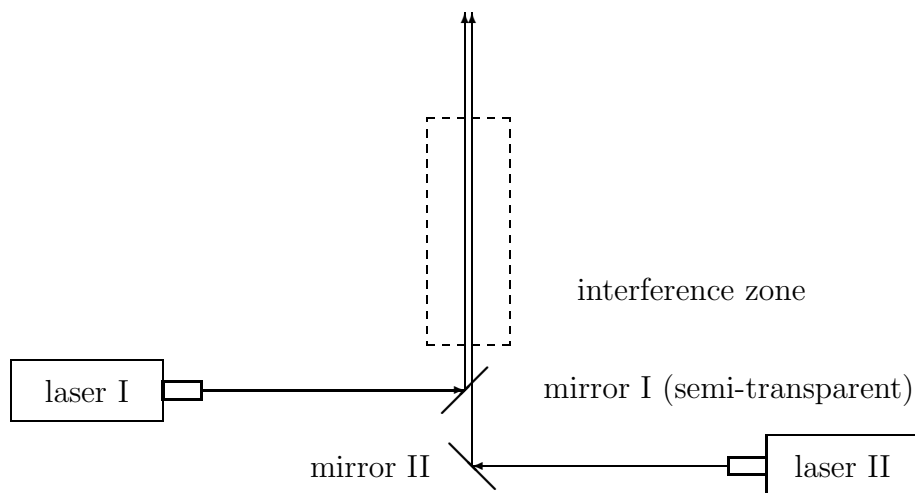


Fig. 2. Principal scheme of the interferometry experiment.

We deliberately exclude from consideration any kind of non-optical experiment and leave beyond our scope the optical experiments in dense matter (water filled columns, etc.). Consider two “identical” sources of monochromatic light which move together in the same direction with the same velocity. The first of them emits a plane wave propagating in the direction of motion of and the second one—in the opposite direction. By means of a mirror and a semi-transparent mirror the two plane waves are made co-linear. The beam of the second laser is reflected by the mirror II changing its direction on  $90^\circ$  and made to pass through a semi-transparent mirror whose reflecting surface serve to change the direction of the beam of first laser on  $90^\circ$ . Beyond the semi-transparent mirror the two beams are parallel and can produce an easily discernible interference striped pattern. A snapshot of the region of interference would reveal strips of different intensity gradually transforming into each other and the modulation frequency can easily be measured.

It is interesting to note that Jaseda *et al.* [34] already used two lasers in interferometry experiment in order to quantitatively verify the FG–L contraction, but in their experiments the lasers beams were parallel (aiming to verify quantitatively the contraction) while in the proposed here experiment they are anti-parallel since now it is not the contraction that needs verification, but the very existence to the first-order of Doppler effect.

For the sake of self-containedness of the paper we outline here the derivation of Doppler effect (see also [35]). The harmonic waves propagating in presumably

quiescent medium are given by the following formula

$$F_{\pm}(x, t) \equiv e^{i(k_r x \mp \omega_r t)}, \quad k_{\pm} = \frac{\omega_r}{c}, \quad \lambda_{\pm} = \frac{c}{\omega_r}, \quad (5.9)$$

where  $\omega_{\pm}$  are the frequencies. The upper sign in the notations refers to the wave propagating in positive direction, while the lower sign—to the wave propagating in negative direction. These waves have to satisfy the boundary condition on the moving boundaries (the sources)

$$F_{\pm}(\pm v_e t, t) \equiv e^{i\omega_0 t}, \quad (5.10)$$

where  $v_e$  is the velocity of the moving frame relatively to the metacontinuum. Respectively, if the sources were at rest, then they would have produced waves with wave number  $k_0 = \omega_0/c$  and wave length  $\lambda_0 = k_0^{-1}$ . The boundary condition (5.10) yields the following relation for the parameters of the propagating wave:

$$\omega_{\pm} = \omega_0 \left(1 \mp \frac{v_e}{c}\right)^{-1} \quad k_{\pm} = \frac{\omega_0}{c} \left(1 \mp \frac{v_e}{c}\right)^{-1} \quad \lambda_{\pm} = \lambda_0 \left(1 \mp \frac{v_e}{c}\right). \quad (5.11)$$

The role of the mirrors is to change the direction of propagation of each wave without destroying its plane nature. After the reflection, the two waves are propagating as planar waves in the positive direction of  $z$ -axis (vertical in Fig. 2):  $F_{\pm}(z, t) \equiv e^{i(k_{\pm} z - \omega_{\pm} t)}$ . Then in the interference region one has a wave which is the superposition of two of them for a given moment of time (say,  $t = 0$ ) so that

$$\Re[F_+(z, t) + F_-(z, t)] = 2 \cos\left(\frac{k_+ + k_-}{2} z\right) \cos\left(\frac{k_+ - k_-}{2} z\right), \quad (5.12)$$

which is a modulated wave with a wave number of the carrier  $\frac{1}{2}(k_+ + k_-) = k_0 + O(d^2)$  and with a wave number for the modulation  $\frac{1}{2}(k_+ - k_-) = dk_0 + O(d^3)$ . Respectively the expressions for wave lengths valid to the second order, are  $\lambda_0$  and  $\lambda_m = \lambda_0 d^{-1}$ . For red-light lasers the length of the wave is  $\lambda_0 \approx 6.3 \cdot 10^{-5}$  cm and then for the length of modulation wave one has  $\lambda_m = 0.63$  cm and the strips produced must be easily detectable on an optical table of standard dimensions.

Alternative way around is to look for the fringes formed on the semi-transparent mirror. The length scale of a fringe would be around 6.3 cm for the red light and the laser beams should be expanded so as to cover a region of 40-50 cm when they reach the semi-transparent mirror.

## 6. Quasi, Pseudo, Meta: Concluding Remarks

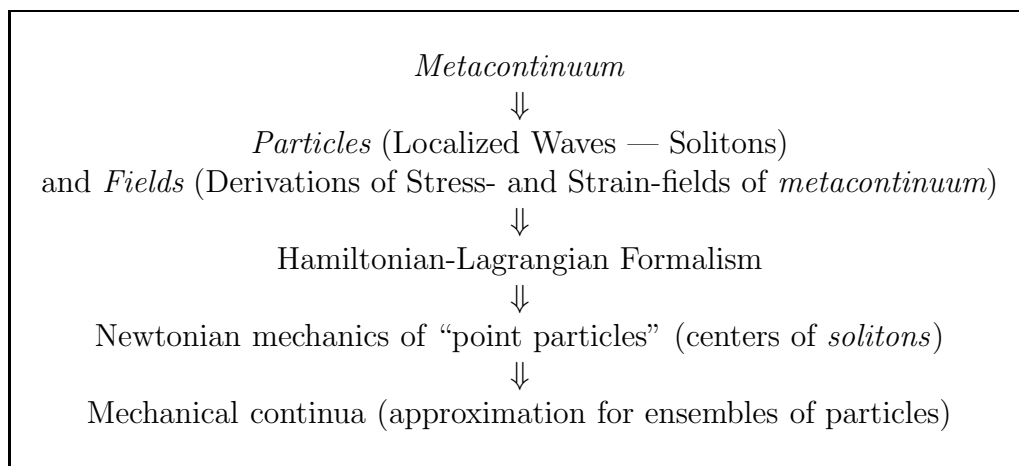
From the point of view of soliton paradigm, the notion of discrete versus continuous (wave–particle dualism) is revisited. As a featuring example a very thin layer of Hookean elastic medium of very large dilational modulus is considered: a special kind of  $N$ -dimensional shell called *gossamer*. It is shown that the linearized equations for the laminar displacements have as a corollary the Maxwell equations for appropriately defined quantities called electric and magnetic fields. A higher-order dispersive and nonlinear Boussinesq equation is derived for the flexural deformations of *gossamer* (“master” equation of the wave mechanics). Its linear part is the linearized Schrödinger equation. Due to the conservative properties of the “master” equation, the flexural solitary waves (*flexons*) appear to be solitons. We call “Soliton Paradigm” the conceptual framework in which the solitons are identified as the particles, i.e. there is no dichotomy between particles and waves. A *particle* (or *corpuscles*) is a notion to signify our perception of the geometry and dynamics of a localized wave (“lump” of deformations, stress, energy, etc.) of *metacontinuum*. The Hamiltonian properties of wave-mechanics equation (the *metadynamics*) define the Hamiltonian (Newtonian) dynamics of the phase objects (*particles-solitons*). Thus the de-Broglie wave-particle duality is a matter of observation and perception: we are faced with a unique object—a localized wave, which is perceived either as a corpuscular object (in fact its center of wave-mass) or as a wave. Particles-Solitons are not moving *through* the metacontinuum, rather they are *phase patterns* propagating *over* it without disturbing its contiguity. Hence no aether-drift effect will be associated with the motion of a particle-soliton. A possible experimental set-up is proposed for verification of this concept.

Solutions of the “master” equation (called *flexons*) are obtained numerically by means of Method of Variational Imbedding. Localized torsional structures of integer-valued topological charge are discussed qualitatively and identified as *charges*. It is shown that the membrane tension in the *gossamer* creates attractive force (*gravitation*) between the large ensembles of localized flexural deflections-particles which is proportional to  $r^{1-N}$ .

The concept of unification based on *metacontinuum* and *soliton paradigm* gives:

- Maxwell Equations for the stress interactions in the middle surface of the *gossamer*;
- Dispersive equation (Schrödinger’s Schrödinger equation) for the wave function of the transverse flexural deformations alongside the fourth spatial dimension;
- Gravitation as membrane (*anti*) tension in the middle surface;
- Charges as vortex-like solutions.

and can be summarized as follows



The unification here is not only for the forces of interaction (the different fields), but it also fuses the Wave-particle duality into Particle-Wave *unity* subordinating the concept of a *particle* (*corpuscle*) to the one of localized phase patterns engendered by the balance between the membrane anti-tension and the nonlinearity and dispersion of the “master” equation of wave mechanics.

*Acknowledgments.* An European Commission Mobility Grant ERBCHBICT 940982 is gratefully acknowledged by the author.

## References

1. E. Whittaker, *A History of the Theories of Aether & Electricity*, Dover, New York, 1989.
2. J. S. Russell, In *Report of 7th Meeting (1837) of British Assoc. Adv. of Sci., Liverpool*, John Murray, London, 1838, p. 417.
3. J. S. Russell, In *Report of 14th Meeting (1844) of the British Association for the Advancement of Science, York*, 1845, p. 311.
4. J. V. Boussinesq, *J. de Mathématiques Pures et Appliquées* **17**(2) (1872) 55.
5. Lord Rayleigh, *Phil. Mag.* **1** (1876) 257.
6. N. J. Zabusky and M. D. Kruskal, *Phys. Rev. Lett.* **15** (1965) 57.
7. C. I. Christov, In *Fluid Physics: Proceedings of Summer Schools*, eds. M. G. Velarde and C. I. Christov, World Scientific, Singapore, 1994, p. 33.
8. T. H. R. Skyrme, *Proc. Roy. Soc.* **A247** (1958) 260.
9. J. K. Perring and T. H. R. Skyrme, *Nuclear Physics* **31** (1962) 550.
10. *Speculations on the Fourth Dimension*, Selected Writings of C. H. Hinton, ed. R.v.B. Rucker, Dover, 1980.
11. T. Kalutza, *Sitz. Preuss. Acad. Wiss.* (1921) S966.

12. O. Klein, *Zeitsch. für Physik* **37** (1926) 895.
13. H. A. Lorentz, Aether Theories and Aether models, In *Lectures on Theoretical Physics*, vol.I, MacMillan, London, 1927.
14. J. C. Maxwell, *Trans. Roy. Soc.* **155** (1865) 469.
15. H. Neuber, *ZAMM* **29** (1949) 97; **29** (1949) 142.
16. M. Dikmen, *Theory of Thin Elastic Shells*, Pitman, Boston, 1982.
17. B. Riemann, *Gesammelte Mathematische Werke und wissenschaftlicher Nachlass*, eds. R. Dedekind and H. Weber, Teubner, Leipzig, 1892, p. 558. (See also English translation in W. Carol, *Energy Potential*).
18. W. K. Clifford, *Mathematical Papers*, ed. R. Tucker, London, 1982.
19. J. L. Ericksen and C. Trussdell, *Arch. Rat. Mech. & Anal.* **1** (1959) 295.
20. E. Schrödinger, *Annalen der Physik* **79** (1926) 743.
21. A. O. Barut, *Foundations of Physics* **20** (1990) 1233; *Phys. Lett. A* **143** (1990) 349.
22. C. I. Christov, In *Proc. 14th Conference of the Union of Bulgarian Mathematicians*, Sunny Beach, Bulgaria, 1985, p. 571.
23. C. I. Christov and M. G. Velarde, *Appl. Math. Modelling*, **17** (1993) 311.
24. C. I. Christov and M. G. Velarde, *Int. J. Bifurc. Chaos* **4** (1994) 1095.
25. S. Sambursky, *The Physical World of the Greeks*, Routledge and Keagan Paul, London, 1963.
26. C. I. Christov and G. A. Maugin, *J. Comp. Phys.* **116** (1995) 39.
27. G. A. Maugin, *J. Mech. Phys. Solids* **29** (1992) 1543.
28. G. A. Maugin, *Material Inhomogeneities in Elasticity*, Chapman and Hall, London, 1993.
29. G. A. Maugin and C. I. Christov, In *Nonlinear Wave Phenomena*, eds. D. Inman and A. Guran, World Scientific, 1996, to appear.
30. D. C. Miller, *Reviews of Modern Physics* **5** (1933) 203.
31. J. C. Maxwell, In *Enciclopediae Britannica*, Ninth Edition, 1875, p. 568.
32. A. A. Michelson, *Phil. Mag.*[5] **13** (1882) 236.
33. A. A. Michelson and E. W. Morley, *Am. J. Sci.* [3] **34** (1887) 333.
34. T. S. Jaseda, A. Javan and C. H. Townes, *Phys. Rev. Lett.* **10** (1963) 165.
35. T. P. Gill, *The Doppler Effect*, Logos Press, 1965.