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Physica A 283 (2000) 156–159

PHYSICA A

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Gradient pattern analysis of Swift–Hohenberg dynamics: phase disorder characterization

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Abstract

In this paper, we analyze the onset of phase-dominant dynamics in a uniformly forced system. The study is based on the numerical integration of the Swift–Hohenberg equation and adresses the characterization of phase disorder detected from gradient computational operators as complex entropic form (CEF). The transition from amplitude to phase dynamics is well characterized by means of the variance of the CEF phase component. © 2000 Elsevier Science B.V. All rights reserved.

PACS: 02.90.+p; 05.45.Pq; 05.45.Tp; 05.70.Ln; 47.20.Ky; 47.54.+r

Keywords: Extended systems; Pattern formation; Gradient dynamics; Complex Entropic form; Phase disorder

Experiments in a variety of areas have shown spatio-temporal complexity, notably in fluid flows, diffusive-reactive systems, optical electronics and plasmas physics (e.g., Ref. [1]). Simultaneously, in the theoretical branch, many studies have analyzed non-linear extended systems and both the emergence and evolution of their spatio-temporal amplitude patterns (e.g., Refs. [2–4]). A typical example may be found in heated Rayleigh–Bernard systems (RBS) and the Swift–Hohenberg equation (SHE) is one of the most important RBS that have been introduced to study spatio-temporal pattern formation [5–8]. The canonical SHE is a dynamical model written as

$$\tau_0 \frac{\partial u}{\partial t} = \varepsilon u - \xi_0^4 (\nabla^2 + k_c^2)^2 u - g u^3 , \quad (1)$$

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PII: S0378-4371(00)00144-8

where τ_0 and ξ_0 are respectively, the natural time and space scales, introduced for dimensional consistency, ε is the bifurcation parameter, k_c is the critical wave number minimizing the critical Rayleigh number, and g is the constant of the nonlinear interaction term.

One interesting feature of SHE is that it contains singular solutions at points that describe the formation of envelope defects from a competition between amplitude and phase dynamics. An important question in this problem concerns the long-term evolution of the patterns, in order to characterize its dynamics not only with regard to the growth of the amplitude of the structure, but also to the evolution of its phase, and then to localize in time the onset of phase dynamics: the main source of defects. In this work, we performed simulations of Eq. (1) in square geometries, starting from random initial conditions with rigid boundary conditions. A sketch of the numerical method may be found in Ref. [10]. As shown by several numerical experiments [8,9] increasing the bifurcation parameter results in a smaller coherence length, which allows the system to develop an envelope with higher density of defects. However, despite the high bifurcation parameter, the emergence of defects is inhibited by small system size. Taking into account this result, the simulations were run with $\varepsilon = 0.50$ in 80×80 square boxes for a system length (and width) of 25. The upper limit of the integration was $t = 2000$. This set of conditions reveals the good qualitative agreement to the most interesting structures obtained by Greenside et al. [6]. Patterns obtained from Eq. (1) using the conditions described above and with $\tau_0 = 0.0509$, $\xi_0^4 = 0.0150$, $g = 1.2900$ and $k_c^2 = 3.1172$ are shown in Fig. 1 (for the simulations using these values we call *canonical conditions simulation*). In Fig. 1a, is shown the field for $t = 2$ when the dominant spatio-temporal dynamics comes from the amplitude growth and in Fig. 1b is shown the field for $t = 240$ when the dominant spatio-temporal dynamics comes from the phase variation. Our goal in this work is to characterize numerically the onset of the phase regime for canonical conditions simulations under small variations of the parameters τ_0, ξ_0^4 and g.

Recently, Rosa et al. [10] have introduced a set of matrix computational operators, for the characterization of asymmetric amplitude fragmentation in nonlinear spatially extended systems. For a given gradient field obtained from the matrix of amplitudes the operators result in a characteristic time series that represents the spatio-temporal evolution of the patterns. For each image the information parameter is a quantitative measure of the structural complexity defined in terms of the degree of asymmetry in the gradient field of the amplitudes. As shown by Rosa et al. [11], the parameters based on the quantity of asymmetric vectors in the gradient field are more robust than spatial correlation length-like operators for characterization of symmetry breaking and phase disorder along the lattice.

In relationship to the pattern changing frequency in the amplitude domain, a complex entropic form (CEF) for the gradient field ∇A is given by

$$S_c(\nabla A) \equiv -\sum \rho_k \log(\rho_k e^{-i\phi_k}) = -\sum \rho_k \log \rho_k - i \sum \rho_k \phi_k ,$$

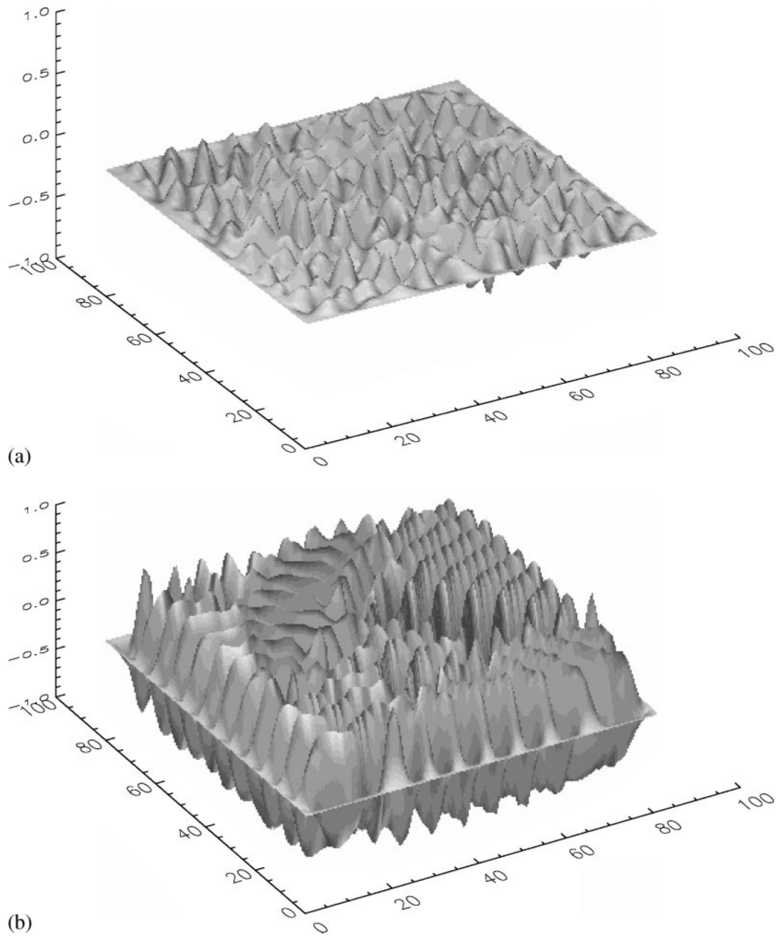

Fig. 1. Two frames of a canonical condition simulation of SHE for (a) $t=2$ and (b) $t=240$.

where ρ_k and ϕ_k are the modulus and the phase of the kth gradient field matrix element and $\mathrm{i}=\sqrt{-1}$. The ρ_k are normalized, so that $\sum \rho_k = 1$, and the ϕ_k are measured from π to $-\pi$. Note that, the $\mathrm{Re}(S_C)$ corresponds to the classical Shannon's entropy measure and that $\mathrm{Im}(S_C)$ represents the weighted average phase of the gradient field. The degree of gradient spatial disorder, can be well characterized by the values of $\mathrm{Im}(S_C)$ over different coarse graining scale lengths, and it is found to be a good measure in the analysis of pattern changing frequency in the amplitude domain [11].

In order to quantify the SH patterns phase disorder, we apply the CEF operator on the simulations of Eq. (1). From the phase component, $\mathrm{Im}(S_c)$, of Eq. (2) we compute its value for the most representative 24-frame set representing the whole simulation using the canonical conditions. The variation of phase disorder parameter along the whole lattice, for each sample, is shown in Fig. 2. The phase variability in the gradient field is characterized by the respective variances (μ_ϕ^2) of $\mathrm{Im}(S_c)$.

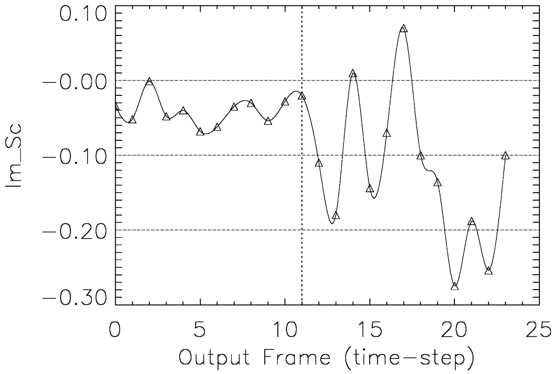

Fig. 2. Evolution of the CEF phase component for a 24-frame set of a canonical condition simulation of SHE.

In a preliminary set of *canonical condition simulations* we performed about 25 simulations varying randomicaly the following parameters: τ_0 and ξ_0^4 from 0.0100 to 0.0200 and g from 1.0000 to 1.5000. For all simulations the variance (μ_ϕ^2) of Im(S_c) have distinguished the amplitude dominant regime from the phase dominant one. We found that $0.0004 < \mu_\phi^2 < 0.0005$ before the transition from the dominant amplitude regime to the dominant phase regime (when μ_ϕ^2 increases about 1000% staying in the range of 0.015–0.017), with the transition ocurring always between frames 11 and 12, that corresponds to $9 < t < 16$ ($\approx$ 0.5% of the whole evolution). This result can be taken as a conjecture to be tested varying also the bifurcation parameter ε and applying analysis based on the characterization of symmetry breaking of the gradient pattern. Work along these lines is in progress and will be communicated later.

The authors RRR, JP, CRN, ELR and FMR are grateful to the agencies FAPESP and CNPq.

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