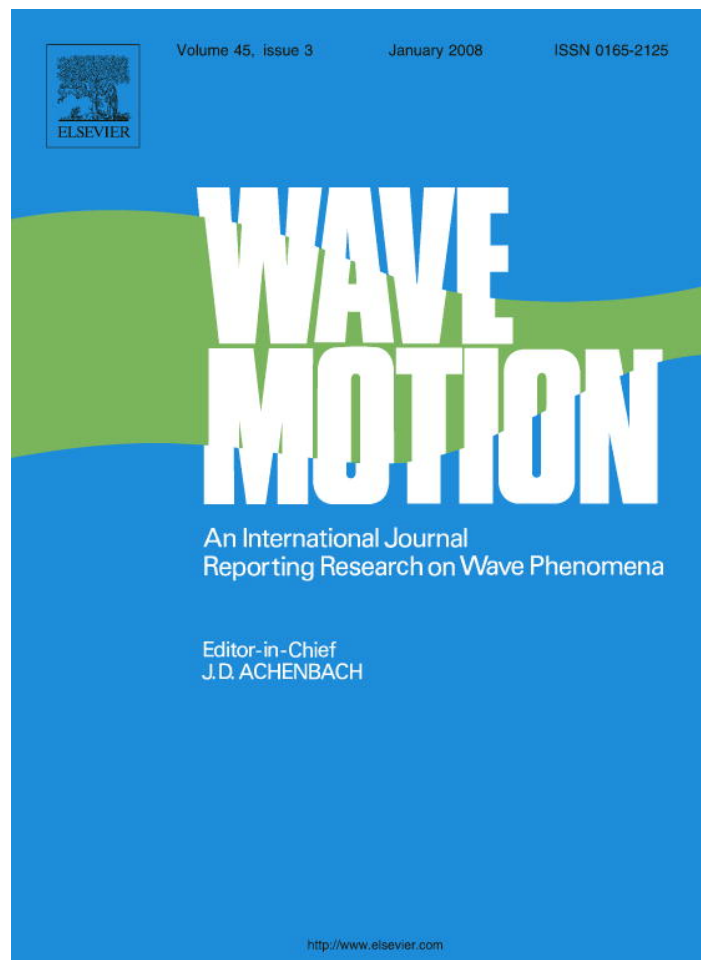


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On the evolution of localized wave packets governed by a dissipative wave equation

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Abstract

The present paper deals with the effect of dissipation on the propagation of wave packets governed by a wave equation of Jeffrey type. We show that all packets undergo a shift of the central frequency (the mode with maximal amplitude) towards the lower frequencies (“redshift” in theory of light or “basshift” in acoustics). Packets with Gaussian apodization function do not change their shape and remain Gaussian but undergo redshift and spread. The possible applications of the results are discussed.

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1. Introduction

The propagation of waves in linear dissipative systems is well studied but most of the investigations are concerned with the propagation of a single-frequency wave. On the other hand, in any of the practical situations, one is faced actually with a wave packet, albeit with a very narrow spread around the central frequency. This means that one should take a special care to separate the effects of dispersion and dissipation on the propagation of the wave packet from the similar effects on a single frequency signal.

The effect of dissipation of the propagation of wave packets seems important because their constitution can change during the evolution and these changes can be used to evaluate the dissipation.

Especially elegant is the theory of propagation of packets with Gaussian apodization function.

2. The model

In this paper we consider the following model equation containing two types of energy loss: absorption and dispersion. In 1D, the model is called Jeffrey’s equation

$$\frac{\partial^2 u}{\partial t^2} + \gamma \frac{\partial u}{\partial t} - \delta \frac{\partial}{\partial t} \frac{\partial^2 u}{\partial x^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad (1)$$

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which has a generic form but it appears in a variety of physically important situations. Naturally, the physical meaning of the coefficients is different in the different models.

The effect of absorption (parametrized by the coefficient γ in Eq. (1)) is to merely attenuate the amplitude of wave motions by the same amount regardless to the frequency. Hence the attenuation does not affect the shape of a wave packet (the apodization function). The effect of dispersive dissipation (parametrized by the dissipation coefficient δ) is different for different frequencies/wave numbers. The dispersion affects the wave packets not only by acting to diminish the total energy of the wave motion but also to change the apodization function of the packet.

One of the oldest applications of Eq. (1) is in the acoustics of viscous compressible fluids. Under the assumption of slight compressibility, the linearized Navier–Stokes equations, continuity equation, and equation of state reduce in 1D to the following system (see, e.g. [1, 2, Chapter 10])

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + \lambda \frac{\partial^2 u}{\partial x^2}, \quad \frac{\partial \rho}{\partial t} + \rho_0 \frac{\partial u}{\partial x} = 0, \quad \frac{\partial \rho}{\partial t} = \frac{1}{c^2} \frac{\partial p}{\partial t}, \quad c^2 = \sqrt{\frac{dp_0}{d\rho_0}},$$

where $u(x, t)$ is the longitudinal component of velocity, ρ is density, and p is pressure. Respectively, λ is the coefficient of bulk viscosity and ρ_0 is the undisturbed density. Note that the coefficient of shear viscosity does not play a role because of lack of shear in 1D acoustics motion (u is not a function of y and z). It is readily shown that density and pressure can be eliminated from the above system to arrive at Eq. (1) with appropriately defined coefficient δ . Note that in classical acoustics, no attenuation is present. The latter appear when porous medium is considered and the Darcy law is added to the N–S equation rendering them 1D to the following

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + \lambda \frac{\partial^2 u}{\partial x^2} - \kappa u,$$

where κ is the inverse of the Darcy coefficient. It is easy to show here that the term proportional to κ results into the attenuation term in Eq. (1). For derivation and discussion, see [3] and the literature cited therein.

We notice here that the bulk viscosity is not the only possible source of dissipation in acoustics. If the heat conduction is acknowledged in the equation of state (the model is called Lighthill’s equation of state), another dissipation mechanism is present that acts in a similar way as the bulk viscosity do.

Another model involving Jeffrey’s equation stems from the theory of viscoelastic continua. A combination of Voigt–Kelvin and Maxwell model (see [4, p. 42], [5, Sec. 7.2]) gives the following relationship between the stress tensor, τ , and the tensor of rate of deformations e

$$\tau + \lambda_1 \dot{\tau} = \eta(e + \lambda_2 \dot{e}),$$

where η is the coefficient of shear viscosity, the dot stands for the full time derivative. Here λ_1 and λ_2 are called “relaxation time” and “retardation time”, respectively. For the 1D shear flow in which $u_x = u_x(y, t)$, one gets the following equation (see [5, Sec. 7.4])

$$\rho \left(\lambda_1 \frac{\partial^2 u_x}{\partial t^2} + \frac{\partial u_x}{\partial t} \right) = \eta \left(\lambda_2 \frac{\partial}{\partial t} \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial y^2} \right), \quad (2)$$

which is once again the Jeffrey’s equation, Eq. (1). Unlike the acoustic case, one has to deal in viscoelastic solids with transverse waves and the physical processes behind the retardation and relaxation times are different from the process in acoustics. Here it is connected with the shear coefficient of viscosity, while in acoustics it is due to bulk viscosity. However, the resulting mathematical model is essentially same.

Another situation in which one arrives at the same generic equation is the two-fluid model of multiphase flows, e.g., the motion of dusty gas. Following [6], the linearized governing equation of the two-fluid model in 1D read

$$\begin{aligned} \frac{\partial u}{\partial t} &= \nu \frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\tau} (u_p - u), \\ \frac{\partial u_p}{\partial t} &= -\frac{1}{\tau} (u_p - u), \end{aligned} \quad (3)$$

where u is the velocity of the continuous phase (the gas), u_p is the velocity of the particulate phase, ν is the kinematic coefficient of shear viscosity of the gas, κ is particle-to-gas density ratio, and τ is a relaxation time connected to the inverse of the Stokes' interaction force between the two phases. By eliminating u_p between the two equations of Eq. (3), one arrives at the following equation, (see [7])

$$\frac{\tau}{1 + \kappa} \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial t} - \nu \frac{\partial^2 u}{\partial y^2} \right) + \eta \left(\frac{\partial u}{\partial t} - \nu \frac{\partial^2 u}{\partial y^2} \right) = 0, \quad (4)$$

which is readily shown to be of the above discussed type Eq. (1).

The above considered models arise also in heat conduction. Replacing the Fourier law by the Maxwell–Cattaneo law (see, [8]) renders the heat conduction equation of hyperbolic type with linear attenuation (see, also [9] for a further discussion of Maxwell–Cattaneo model). A rather similar situation to the theory of dusty gas arises in microscopic heat conduction when two temperature fields are considered (see [10] for details).

The list of physically relevant models which reduce in 1D to Jeffrey's equation Eq. (1), can easily be extended further but it goes beyond the scope of the present work to give an exhaustive survey. The above mentioned examples suffice to claim that investigating the propagation of waves as governed by Eq. (1) is of importance. When single-frequency modes are concerned, exhaustive studies can be found in the literature. The purpose of the present work is to elucidate the specific properties of propagation of wave packets as contrasted to the properties of propagation of single-frequency modes.

3. Propagation of spatial modes: dispersion relation

Consider harmonic waves $e^{ikx + i\hat{\omega}t}$, where $\hat{\omega}$ can be, in general, a complex valued variable. The dispersion relation for the propagation of harmonic waves governed by Eq. (1) is

$$-\hat{\omega}^2 + i\gamma\hat{\omega} + i\delta\hat{\omega}k^2 = -c^2k^2, \quad (5)$$

which can be solved for the complex frequency $\hat{\omega}$ as function of the real wave number k

$$\hat{\omega} = i\frac{\gamma + \delta k^2}{2} \pm \sqrt{c^2k^2 - \left(\frac{\gamma + \delta k^2}{2}\right)^2}. \quad (6)$$

It is easily seen that for

$$c|k| < \left(\frac{\gamma + \delta k^2}{2}\right), \quad (7)$$

the discriminant is negative which means that $\hat{\omega}$ is strictly imaginary (no real part). In this case the system is over-damped and no oscillations in time are possible. This means that no traveling modes can exist in that limit, and then the quantity $i\hat{\omega} = -\frac{1}{2}(\gamma + \delta k^2)$ defines the attenuation of the amplitude of the standing wave with time. First we mention that for $\delta\gamma > c^2$ the condition (7) is satisfied and the discriminant is always negative which means that no oscillations in time are possible. However, even if $\delta\gamma < c^2$, the interval of existence for oscillations is limited to

$$\frac{c - \sqrt{c^2 - \gamma\delta}}{\delta} = k_0 < k < k_\infty = \frac{c + \sqrt{c^2 - \gamma\delta}}{\delta}, \quad (8)$$

and no oscillations in time can exist outside this interval. This means that only standing modes are possible outside that interval.

The case $\delta\gamma \ll c^2$ is of practical importance, because the speed of sound or light is usually much larger than the attenuation and dissipation coefficients. For this case, the above expression, Eq. (8) can be simplified to

$$\frac{\gamma}{2c} < \frac{\gamma}{2c - \frac{\delta\gamma}{2c}} < k < \frac{2c}{\delta} - \frac{\gamma\delta}{2c} < \frac{2c}{\delta}, \quad (9)$$

i.e., the traveling modes cannot exist for

$$k < \frac{\gamma}{2c} \quad \text{or} \quad k > \frac{2c}{\delta}.$$

The lower cut-off frequency is governed by the attenuation, while the upper cut-off frequency is governed by the dissipation. In other words, for finite values of the attenuation coefficient, γ , very long waves are impossible, and for finite value of dissipation coefficient, δ , the interval is limited from above which means that highly oscillatory modes are impossible. When $\delta \rightarrow 0$, $\kappa \rightarrow 0$, the interval of existence of traveling waves extends to $[0, \infty]$.

4. Localized wave packets

Let us consider here a wave packet for which the initial apodization function $E(k, 0) \in L^2(-\infty, \infty)$ gives the amplitude of the respective Fourier mode as function of the wave number k . The actual profile of the wave packet as function of the spatial variable x is given by the Fourier integral

$$\int_{-\infty}^{\infty} e^{ikx} E(k, 0) dk. \tag{10}$$

Any localized apodization function satisfying the condition

$$\int_{-\infty}^{\infty} E^2(k, 0) dk < +\infty,$$

can represent a wave packet. If the spectral distribution $E(k, 0)$ is taken as an initial condition, the spectral content of the packet after time t be given by

$$E(k, t) = E(k, 0) e^{i\omega(k)t} e^{-s(k)t}, \tag{11}$$

where $\omega = \omega(k)$ is the actual (real valued) frequency of the wave, and $s = s(k)$ gives the decay factor for the wave of given wave number. These are given by the formulas

$$\hat{\omega} = \omega + is, \quad \omega = \sqrt{c^2 k^2 - \left(\frac{\gamma + \delta k^2}{2}\right)^2}, \quad s = \frac{\gamma + \delta k^2}{2}, \tag{12}$$

Eq. (12) shows that an initial distribution of the energy as function of k will change in time in the sense that the amplitudes of the shorter waves will diminish faster in time than the amplitudes of the longer waves. This will lead to redistribution of the amplitudes and to a change of the apodization function of a wave packet that is subject to evolution according to Jeffrey's equation. Therefore a general shift of the central wave number towards longer waves (smaller wave numbers k) is to be expected. In the case of light, this is called "redshift". The quantitative values for the redshift for different apodization functions may differ. The most interesting case appears to be the Gaussian distribution of the packet and we focus in this short note on the said case.

4.1. Gaussian apodization function

A Gaussian apodization function is given by

$$E(k, 0) = e^{-\frac{1}{2}\beta(k-\tilde{k})^2}, \tag{13}$$

where \tilde{k} is the central wave number of the packet, and the coefficient β is related to the inverse of the width of the wave packet. A larger β means a sharper spectral line in terms of wave number k , or what is equivalent, a broader spectral line in terms of spatial frequency, $\lambda = 2\pi k^{-1}$. In general $\beta = \beta(\tilde{k})$. A constant β means that the width of the spectral line under consideration is not related to the value \tilde{k} of the central wave number.

Let us denote by q the value of wave number k for which the amplitude defined by Eq. (13) is exactly $e^{-\pi/4}$ times smaller than the maximal amplitude at $k = \tilde{k}$. If we introduce the notation

$$d = \frac{q - \tilde{k}}{\tilde{k}},$$

for the “width” of the wave packet, then the above requirement for the amplitude $E(q)$ is satisfied when

$$d = \sqrt{\frac{\pi}{2\beta}} \quad \text{or} \quad \beta = \frac{\pi}{2d^2}. \quad (14)$$

Hence, the total area under the curve from Eq. (13) (the integral of $E(k)$ over the entire interval) is $A = \sqrt{2\pi/\beta}$, then the area of a rectangle of base d and height one is $d = \sqrt{\pi/(2\beta)} = \frac{1}{2}A$. This means that Eq. (14) gives the effective half-width of the packet.

Now, consider an initial condition given by the following Fourier integral

$$u_0(x) = \int_{-\infty}^{\infty} e^{-\frac{1}{2}\beta(k-\tilde{k})^2} e^{ikx} dk. \quad (15)$$

Then the evolution of Fourier density of the wave packet is given by the following simple relation

$$E(k, t) = e^{i\omega t + ikx} e^{-\frac{1}{2}\beta(k-\tilde{k})^2} e^{-\frac{1}{2}(\gamma + \delta k^2)t}, \quad (16)$$

for the amplitude of a component with specific wave number k . The first term in Eq. (16) gives a traveling wave (when it exists) with phase velocity

$$\frac{\omega}{k} = c \sqrt{1 - \left(\frac{\gamma k^{-1} + \delta k}{2c} \right)^2}. \quad (17)$$

4.2. Redshifting of Gaussian localized packets

Let us examine closely the shape of the spectrum as given by Eq. (11), (or by Eq. (16) for the Gaussian case). We limit ourselves in this subsection to the case of Gaussian apodization function in order to elucidate the main idea of how the dissipation influences the constitution of a wave packet. It is readily seen that for $t = T > 0$, the maximum of $E(k, T)$ is not anymore at $k = \tilde{k}$. The position of the maximum is given by the condition

$$\frac{dE(k, T)}{dk} = [-\beta(k - \tilde{k}) - \delta Tk] e^{-\frac{1}{2}\beta(k-\tilde{k})^2} e^{-\frac{1}{2}(\gamma + \delta k^2)T} = 0,$$

which is a linear equation for k and its root gives the following value \bar{k} at which the maximum is situated after time T has passed

$$\bar{k} = \frac{\beta\tilde{k}}{\beta + \delta T} = \frac{\tilde{k}}{1 + \delta\beta^{-1}T}. \quad (18)$$

For the shift of the central wave number (the wave number for which the amplitude of packet is maximal) we get

$$\Delta k \stackrel{\text{def}}{=} \bar{k} - \tilde{k} = -\frac{\tilde{k}\delta\beta^{-1}T}{1 + \delta\beta^{-1}T} \quad \text{or} \quad \bar{k} = \frac{\tilde{k}}{1 + \delta\beta^{-1}T}. \quad (19)$$

Then

$$\frac{\Delta k}{\tilde{k}} = -\frac{\delta\beta^{-1}T}{1 + \delta\beta^{-1}T} \quad \text{or} \quad \frac{\Delta k}{k} = -\delta\beta^{-1}T, \quad (20)$$

which means that at time T , the central wave number \bar{k} of a packet is shifted to the smaller wave numbers in comparison with the initial central number \tilde{k} . In terms of wave length $\lambda = 2\pi/k$, the wave number is expressed as $k = \pi/\lambda$ and we get for the shift that

$$\Delta k = \left(\frac{2\pi}{\lambda} - \frac{2\pi}{\tilde{\lambda}} \right) = -2\pi \frac{\lambda - \tilde{\lambda}}{\lambda \tilde{\lambda}} = -\frac{2\pi}{\tilde{\lambda}} \frac{\Delta \lambda}{\lambda} = -\tilde{k} \frac{\Delta \lambda}{\lambda},$$

which gives

$$z = \frac{\Delta \lambda}{\tilde{\lambda}} = -\frac{\Delta k}{\tilde{k}} = \delta \beta^{-1} T, \tag{21}$$

where we use the ubiquitous notation for the redshift, z .

The last expression means that the central wave length of the packet is shifted to the longer waves. Borrowing a coinage from the theory of light we can say that the packet is redshifted. Naturally, for acoustic waves it should be called *baseshift*. Eq. (21) shows that the redshift depends *linearly* on the time of propagation. As witnessed by the Eq. (17), the group velocity of the packet is not constant. However, for very tightly localized packets, we can assume that the group velocity is well approximated by the phase velocity $c_g = \omega(\tilde{k})/\tilde{k} = c$ and to assess the propagation time as $T \approx c^{-1}L$, where L is the length traveled by the packet. Then Eq. (21) adopts the form

$$z = \frac{\Delta \lambda}{\tilde{\lambda}} = \frac{\delta}{c\beta} L = HL, \quad \text{where } H = \frac{\delta}{\beta c}. \tag{22}$$

The relation Eq. (22) can be called “Hubble law” for redshifting of a Gaussian wave packet. More precisely we shall call it “distance-Hubble law”, because the redshift is proportional to the distance traveled by the wave packet. Then Eq. (21) can be named “time-Hubble law”. Note that due to the intricate dependence of the group velocity on wave number, the time-Hubble law is strictly linear, while the distance-Hubble law can be slightly nonlinear because the Hubble constant can be function of \tilde{k} through c_g and hence of distance L for a given T . For $\gamma, \delta \ll c$ one gets that $c_g \approx c$ which means that the distance-Hubble law is also virtually linear. Even if we assume that $\gamma, \delta \ll c$, Hubble constant is a function of the central wave number \tilde{k} through the dependence $\beta = \beta(\tilde{k})$ which specifies the width of a packet as a function of its central wave number.

It is useful to rewrite Eq. (11) as follows

$$E(k, T) = e^{-\frac{\gamma}{2}T} e^{-\frac{1}{2}(\beta + \delta T)(k - \tilde{k})^2} e^{-\frac{1}{2}\delta T(1 + \delta\beta^{-1}T)^{-1}}, \tag{23}$$

which means that the actual magnitude of the spectrum at its maximum is

$$E_m(T) = \max_k E(k, T) = e^{-\frac{\gamma}{2}T} e^{-\frac{1}{2}\delta T \tilde{k}^2 (1 + \delta\beta^{-1}T)^{-1}}.$$

The magnitude of packet’s maximum decays exponentially with time with an exponent that depends linearly on the attenuation γ , but the role of the dispersive dissipation δ is more intricate. For very large times T , the role of dissipation amounts to an additional attenuation $\beta/2$ which means that a packet that is narrower in terms of wave number (wider in wave length) is attenuated more. Clearly, for non-zero values of T , the magnitude of the maximal amplitude decreases exponentially. It is convenient to scale the wave packet by $E_m(T)$ and to concern ourselves with the renormalized spectrum

$$\hat{E}(k, T) = \frac{E(k, T)}{E_m(T)} = e^{-\frac{1}{2}(\beta + \delta T)(k - \tilde{k})^2}, \tag{24}$$

which is once again a Gaussian distribution with a larger coefficient β , namely

$$\beta^T = \beta + \delta T \equiv \beta(1 + z). \tag{25}$$

The main conclusion from this subsection is that the dissipation acts to shift the central wave length of the packet to the longer waves and to increase its width while preserving the Gaussianity. Preserving the shape of the spectral density is a unique property of the Gaussian apodization functions.

4.3. Spatial propagation

In this section we consider the propagation of waves from a boundary condition at $x = 0$ which is a superposition of harmonic waves of type $e^{i\omega t}$, namely $\int E(\omega) e^{i\omega t} d\omega$. We can rewrite the dispersion equation as

$$k^2 = \frac{\omega^2 - i\gamma\omega}{c^2 + i\delta\omega},$$

which gives for the imaginary part of the wave number (responsible for the spatial growth or decay of the wave) the following

$$\Im[k] = \pm \frac{1}{\sqrt{2(c^4 + \delta^2\omega^2)}} \sqrt{-(c^2 - \gamma\delta)\omega^2 + \sqrt{(c^2 - \gamma\delta)^2\omega^4 + (\gamma\omega c^2 + \delta\omega^3)^2}}. \quad (26)$$

The sign “+” will not be considered because we are interested in waves that propagate to the right and decay at $x \rightarrow \infty$. The relation, Eq. (26), can be simplified under the above adopted condition $\gamma\delta \ll c^2$. We also assume $(\gamma\omega c^2 + \delta\omega^3) \ll c^2\omega^2$ and then Eq. (26) is replaced by

$$\Im[k] \approx \frac{1}{2} \frac{\gamma c^2 + \delta \omega^2}{c^3}. \quad (27)$$

A packet produced at $x = 0$ with a Gaussian distribution (the factor c^2 is introduced, to make coefficient β have the same dimension as in previous sections)

$$E(\omega) = e^{-\frac{1}{2}\beta c^2(\omega - \bar{\omega})^2}, \quad (28)$$

will arrive at $x = L$ with the following constitution

$$e^{-\frac{1}{2}\beta c^2(\omega - \bar{\omega})^2} e^{-\frac{1}{2}(\gamma c^2 + \delta \omega^2)c^{-3}L} = e^{(-\frac{1}{2}\gamma - \delta \bar{\omega} \bar{\omega})c^{-1}L} e^{-\frac{1}{2}\beta c^2(\omega - \bar{\omega})^2},$$

where the central frequency is changed to

$$\bar{\omega} = \frac{\tilde{\omega}}{1 + \delta\beta^{-1}c^{-1}L}. \quad (29)$$

This means that the redshifts for the frequency ω and the wave length $\lambda = \frac{2\pi}{\omega}$ are, respectively

$$\frac{\Delta\omega}{\bar{\omega}} = -\frac{\delta\beta^{-1}c^{-1}L}{1 + \delta\beta^{-1}c^{-1}L}, \quad \frac{\Delta\lambda}{\tilde{\lambda}} = \delta\beta^{-1}c^{-1}L. \quad (30)$$

Now the distance-Hubble law is linear with Hubble constant $H = \delta\beta^{-1}c^{-1}$. One can easily recast Eq. (30) as a time-Hubble law through the relationship $T = Hc_g^{-1}$, which can be nonlinear.

5. Conclusions

In the present work, the effect of attenuation and dissipation on propagation of waves governed by the Jeffrey equation is addressed. When packets of small but finite breadth are considered the presence of dissipation changes the central wave number of the packet. The distribution of the wave length around the central length is assumed to be Gaussian which is the most frequently encountered case in cosmology when hot stars are observed. Dispersion relation for the damped wave equation is derived and the evolution of the packet density is investigated in time (or space). It is shown that the attenuation acts merely to decrease the amplitude of the shifts packed, while the dissipation damps the higher frequencies stronger than the lower frequencies and shifts the maximal frequency of the packet to lower frequencies (longer wave lengths), i.e., the packet appears redshifted upon its arrival. For Gaussian wave packets, this kind of redshift is linearly proportional to the time passed or the distance traveled. The coefficient of proportionality contains the ratio of the dissipation coefficient and the initial width of the distribution which means that the thicker packets are redshifted more than the narrower ones for the same distance or for the same time. We call this liner relationship “Hubble Law” for redshifting of wave packets.

The new approach can be used in acoustics for devising methods for estimating the bulk viscosity of air, or other slightly compressible liquids based on the relationship between the “baseshift” and viscosity coefficient. An application to cosmology is also possible because the spectral lines measured in the experiments are wave packets, and never a single isolated wave comprising it. Thus, one has to take special care to distinguish

between the redshift of the packet (as outlined in the present work) and the redshift due to the dilation of a single wave.

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