Beat wave interferometry for measuring relative motion

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A new approach to detecting relative motion is proposed based on measuring the beat frequency. A formula is derived for the beat frequency as a function of the speed of translation. A principal sketch of an experimental setup implementing the new idea is proposed, and the effects of possible small differences in the frequencies and phases of the sources are discussed. © 2009 Optical Society of America
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1. INTRODUCTION

Interest in measuring the speed of Earth relative to the surrounding space was rekindled after the anisotropy of the cosmic background radiation (CMBR) was discovered. The presence of anisotropy in the CMBR clearly defines the relative speed of Earth of the order of 240–360 km/s (see [1,2]). Since the predominant belief is that there exists no absolute medium where light propagates, this relative speed is referred now to as the speed of the local standard of rest (LSR). The precise measurement of this speed is of vital importance for synchronization of the clocks on GPS satellites orbiting the Earth. There is no doubt that it is of paramount importance to obtain independent confirmation of the results regarding the relative speed; regardless of the terminology (e.g., "preferred framework" and "local standard of rest" versus "absolute medium" and "physical vacuum"), the problem of detecting the relative motion is still on the agenda.

Measuring the relative speed in a framework connected to Earth is facing the same difficulties that plagued the scientific community more than a century and a quarter ago, when Michelson undertook the seemingly impossible task of measuring a vanishingly small effect. The difficulty was first pointed out by Maxwell [3]: it was connected with the fact that the effect of the relative motion on the interference of a light beam following a closed circuit is inevitably of second order with respect to the ratio of the relative speed to the speed of light, $O(u^2/c^2)$.

To measure the speed v of the relative motion, Michelson chose to use what can be called phase interferometry, which has been the main technique used since the groundlaying work of Fizeau (see the overview in [4]). The idea of phase interferometry is to quantify the time difference for the beams that travel different paths through observing the fringes that are produced after superposition of the beams. The effect of the relative speed was expected to be apparent from the changing position of the fringes and/or the number of fringes per unit length. Since no appreciable change in the fringes was observed in the Michelson and Morley experiment, it was said that it produced a nil result.

Fitzgerald (see [5], p. 749) and Lorentz [6] explained the nil effect by the possible contraction of the lengths in the direction of motion. The Lorentz–Fitzgerald contraction has been splendidly confirmed in all major experimental tests and can now be considered to be one of the most important discoveries in physics. Accepting the contraction assumption instantly renders phase interferometry incapable of detecting the relative motion. This means that every effort must be made to create an experimental setup that allows measurement of the relative speed in a laboratory setting.

Clearly, an experiment dealing with the relative speed has to be based on a superposition of beams that travel differently through the resting medium (or the LSR). Nowadays, with the advent of highly stabilized lasers, creating almost identical sources in different spatial positions becomes a real possibility. Such an approach can be termed "two-beam interferometry" (TBI). TBI implements the most natural idea of interferometry: to detect the beat frequency, which should arise after the superposition of two beams whose wavenumbers are changed differently by the Doppler effect. The idea of using two sources of light in interferometry to create a beat frequency was first floated in [7]. After the issues of the reflection from the moving mirrors were clarified in [8], the scheme of the experiment was modified in order to avoid the cancellation of the Doppler effect. The most comprehensive exposition of mathematics behind TBI can be found in [9].

TBI faces formidable difficulties connected with the long-time drift of the laser frequencies ("coherent length"). In order to alleviate these difficulties, we propose in the present paper an experimental scheme that is based on reflection from a mirror. We adapt the theory of beat frequency to this case and propose a possible experimental setup.

2. BEAT FREQUENCY IN THE INTERFERENCE OF TWO OPPOSITE BEAMS FROM CO-MOVING SOURCES

Assume now that an electromagnetic wave is excited at a given point that is moving together with a mirror in the same direction (say, from left to right). The wave from the left source propagates in the direction of the motion (to the right), while the wave reflected from the mirror source propagates in the opposite direction (to the left). As argued in [8], the effect of the fact that the mirror is co-moving with the source of light relative to the LSR cancels the original emitter's Doppler effect, and after the reflection we get the emitter's Doppler effect merely from the fact that the mirror is receding relative to the LSR.

Assume, for definiteness, that the mirror provides a virtually perfect reflection, which means that the amplitude of the reflected wave is virtually the same as the impinging wave. Denote by f the respective component of either the electric or the magnetic vector. Then the interference between the right-going wave from the left source and the left-going wave from the right source (accounting also for the Doppler effect) is given by

$$\begin{aligned} f(x,t) &= \cos\left[\omega\left(\frac{x}{c}-t\right)\Big/\left(1-\frac{u}{c}\right)\right] \\ &\quad + \cos\left[\omega\left(t+\frac{x}{c}\right)\Big/\left(1+\frac{u}{c}\right)+\theta\right] \\ &= 2\cos\left[\left(-\hat{\omega}t+\tilde{\omega}\frac{x}{c}\right)+\frac{1}{2}\theta\right] \\ &\quad \times\cos\left[\left(-\tilde{\omega}t+\hat{\omega}\frac{x}{c}\right)-\frac{1}{2}\theta\right], \end{aligned}$$

$$\tilde{\omega}=\omega\left(1-\frac{u^2}{c^2}\right)^{-1}, \quad \hat{\omega}=\frac{u}{c}\tilde{\omega}. \tag{1}$$

Here θ is the phase shift due to the reflection. The influence of the mirror's motion on the phase of the reflected wave is still not well elucidated in the literature. However, as will be shown later in this paper, the actual phase does not really matter for the proposed method of measurement. The situation in which the beat frequency can be created is presented in Fig. 1.

The last term on the r.h.s. of Eq. (1) gives the carrier wave with frequency slightly deviating from the main frequency, while the first term on the r.h.s of Eq. (1) is the modulation wave, whose frequency is a fraction of the main frequency proportional to u/c. This means that the modulation frequency is related to the first-order Doppler effect in the medium, and its measurement can give a quantitative estimate for the speed of the relative motion.

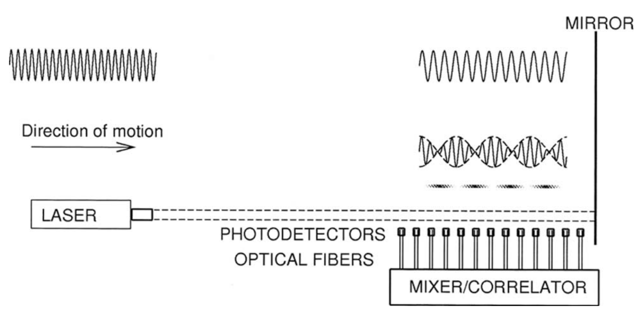

Fig. 1. Experimental setup involving two lasers/masers.

3. ROBUSTNESS OF THE EFFECT TO SMALL IMPURITIES OF THE FREQUENCIES AND PHASES OF THE LIGHT SOURCES

It is accepted nowadays that the speed of the LSR, to which the solar system belongs, is of the order of several hundred kilometers per second relative to the center of the local cluster of galaxies [1,2,10]. We can safely assume that $v\approx 300$ km/s, which gives $\varepsilon\equiv u/c\approx 10^{-3}$. For completeness, we mention that the lowest value for ε is 10^{-4} based on the orbital speed of Earth if there is no motion of the solar system with respect to the vacuum. Thus, the range for ε to be targeted in an experiment such as the one proposed here is $\sqrt{10^{-3}}\gtrsim\varepsilon\gtrsim 10^{-4}$.

Note that we can also neglect here the phase difference θ between the two lasers, because the latter simply acts to displace the carrier and beat waves in space (time) without having any effect on the quantitative value of the beat frequency (wavenumber). This is a crucial advantage of beat wave interferometry over classical Michelson phase interferometry.

Before proceeding further, we should emphasize the fact that the above-described modulated wave is excited *in the* surrounding *resting* medium, and not in the moving frames. In most of the theoretical works on the subject, this fact is very often left without comment. It is important to understand that the light is emitted by elements of the moving frame but consequently "detaches"' from the latter and propagates in the absolute medium. Then it is captured again by sensors in the moving frame, which process is subject to the receiver's Doppler effect. Since the entire setup is moving with speed u in the positive direction along the x axis, then we have to change to a moving frame in order to get a "reading" of the pattern that is created in the absolute medium; in other words, we will have the receiver's Doppler effect at the detecting screen. To elucidate this point, we consider the moving frame $x=\xi+ut$ and render Eq. (1) as the following:

$$f(\xi,t)=2\cos\left[\tilde{\omega}\left(1-\frac{u^2}{c^2}\right)t-\hat{\omega}\frac{\xi}{c}\right]\cos\left(\hat{\omega}\frac{\xi}{c}\right), \tag{2}$$

which is a fast-moving envelope over a standing wave.

The difficulty stems from the fact that the group speed of the envelope $c\tilde{\omega}/\hat{\omega}\approx c^2/u$ is very large (much larger of the speed of light), and an instantaneous snapshot can be informative only if the wave did not move appreciably in the spatial direction and did not smear the beat pattern. It is well known that the group velocity can be larger than the speed of light without contradicting the main postulate that the speed of light is the limiting speed for material processes. Clearly, it is impossible to create a camera with such a quick shutter without violating the mentioned postulate. A feasible approach to the detection is to measure a two-point correlation. If one places several photodetectors in the area of the interaction of the two beams, one can get the correlation by time averaging the product of the two amplitudes measured. Actually, there will also be a spatial averaging because of the size of the photodetector. This size has to be large enough in comparison with the wavelength of the carrier frequency and small enough in comparison with the wavelength of the beat wave. For visible light this places the dimension of the photodetectors at $d \approx 10\ \mu$m. The same number gives a good estimate of the distance between the different photodetectors.

The above argument can be formalized if one introduces the following average procedure:

$$\langle \Phi(x,t) \rangle = \int_0^T \int_0^d \Phi \mathrm{d}x \mathrm{d}t. \tag{3}$$

This kind of averaging will filter the highly oscillatory spatial patterns related to the carrier wave and the temporal undulations due to the propagation speed of the envelope. Then for the correlation of the signal between two photodetectors separated by a distance z, we get

$$K(z) = \langle f(\xi + z, t) f(\xi, t) \rangle = B \cos\left(\frac{uz}{c}\right), \tag{4}$$

where B is a constant. From the profile of the correlation, one can identify the period L of the cosine function $\cos(2\pi z/L)$ that fits it best. From L, the relative velocity of the moving frame is identified as

$$u = \frac{2\pi c^2}{2L\omega} + O(\varepsilon^2) \approx \frac{c\lambda}{2L}. \tag{5}$$

For instance, if we use red-light lasers with wavelength $\lambda \approx 600$ nm and we measure a spatial period $L \approx 0.3$ mm, then for the relative speed, we get $u \approx 300$ km/s. The same result will be reached if one observes strips of width 0.6 mm when using an IR laser with wavelength 1200 nm.

4. POSSIBLE EXPERIMENTAL SETUP FOR MEASURING THE FIRST-ORDER DOPPLER EFFECT

The beat frequency is expected to be about 10^{-3} times smaller than the carrier frequency, putting it on the order of several hundred gigahertz. Apart from the fact that mirrors were used in [11,12] (see the discussion about the reflection from moving mirrors in [8]), the high frequency of the beat wave could have been another reason why it was not detected as an unwanted disturbance in those experiments. In fact, Townes *et al.* [11,12] were after the much lower beat frequency connected with the second-order effects and found practically no beat. The same experiment was further refined in [13], increasing the sensitivity 4000 times, and no beat was found. This is exactly what is to be expected in light of the discussion in [8], where the claim was made that no effect (first-, second-, or higher-order) of the beat frequency can exist if reflections from *moving* mirrors are involved.

The way to conduct the experiment free of the above described difficulties connected with measuring the time frequency at a given spatial point is to measure the wavenumber $\hat{k}$ of the spatial beat wave by taking a snapshot of the wave at a certain moment in time. A possible experimental setup implementing this idea is presented in Fig. 1. In the proposed scheme, one is assumed to observe a spatial correlation as given by Eq. (4). The expected pattern is shown in the upper-center part of the figure.

It should be noted here that the inevitable small drift in time of the laser frequencies should not affect the result obtained from an instantaneous snapshot. Thus, no effects connected with the rate of frequency drift are expected. The actual value added to the frequency due to the drift is usually so small that it cannot change the spatial distribution of the intensity during a snapshot.

5. CONCLUSION

A radically new type of interferometry experiment is proposed based on measuring the spatial wavenumber of the beat wave that occurs when a beam and its reflection from a moving mirror interfere. It is shown that the spatial wavenumber of the beat wave is proportional to the ratio of the relative speed and the speed of light. Thus, an actual situation is found in which the first-order Doppler effect can be measured.

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