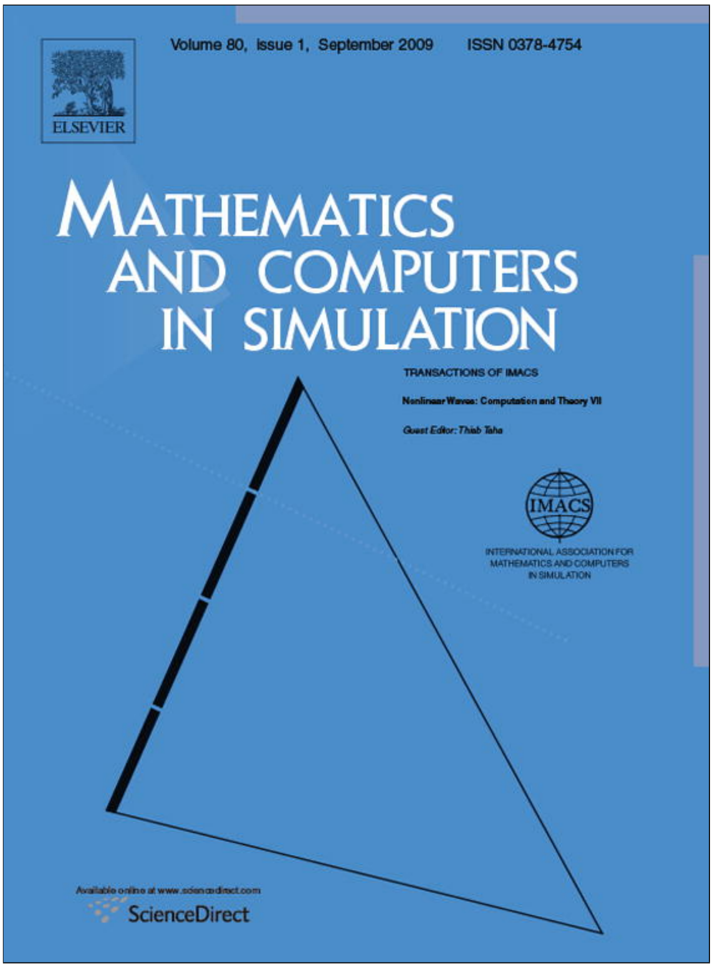
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The concept of a quasi-particle and the non-probabilistic interpretation of wave mechanics

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Abstract

In recent works of the author [*Found. Phys.* 36 (2006) 1701–1717; *Math. Comput. Simul*. 74 (2007) 93–103], the argument has been made that Hertz's equations of electrodynamics reflect the material invariance (indifference) of the latter. Then the principle of material invariance was postulated *in lieu* of Lorentz covariance, and the respective absolute medium was named the *metacontinuum*.

Here, we go further to assume that the *metacontinuum* is a very thin but very stiff 3D hypershell in the 4D space. The equation for the deflection of the shell along the fourth dimension is the "master" nonlinear dispersive equation of wave mechanics whose linear part (Euler–Bernoulli equation) is nothing else but the Schrödinger wave equation written for the real or the imaginary part of the wave function. The wave function has a clear non-probabilistic interpretation as the *actual* amplitude of the flexural deformation.

The "master" equation admits solitary-wave solutions/solitons that behave as quasi-particles (QPs). We stipulate that particles are our perception of the QPs (*schaumkommen* in Schrödinger's own words). We show the passage from the continuous Lagrangian of the field to the discrete Lagrangian of the centers of QPs and introduce the concept of (pseudo)mass. We interpret the membrane tension as an attractive (gravitational?) force acting between the QPs. Thus, a self-consistent unification of electrodynamics, wave mechanics, gravitation, and the wave-particle duality is achieved.

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1. Introduction

The wave phenomenon is associated with either the transverse or longitudinal elastic vibrations of a certain medium (field). If there is a wave, something material should be waving. This notion led 19th century scientists to introduce the concept of the luminiferous continuum (field, aether, etc.). Cauchy explained the Fresnel experiments by modelling the aether as an elastic continuum. Lord Kelvin considered the particles as vortices in the aether. Maxwell and Hertz treated light as a wave phenomenon of the electromagnetic field (again the concept of continuum!).

Soon after Maxwell formulated his equations, it was discovered that his model was not invariant with respect to translational motion of the frame. Hertz (*circa* 1895) realized that the cause of non-invariance was the use of partial time derivatives. He proposed to use the material (convective) time derivative *in lieu* of partial time derivatives. Unfortunately, Hertz's proposal did not attract much attention, and the search for a substitute for the material invariance continued.

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Voigt, and independently Lorentz (see [16]), spotted the fact that the wave equation can be made invariant, if the moving frame time variable is changed together with the spatial variables. Nowadays, this is known as the Lorentz transformation. The success of the latter stems from the fact that it tacitly restores some terms of the convective derivative, i.e., it emulates the material invariance for non-deformable frames in rectilinear motion [9]. Although people speak about general (Lorentz) covariance, it has to be pointed out that the Lorentz transformation has no meaning for accelerating frames, hence Lorentz covariance can be called "*Poor Man's Material Invariance*" [9] and the search for the true invariant formulation should continue.

2. Rational metamechanics of the luminiferous medium (special theory of absolutivity?)

The electromagnetic field resembles both elastic (Cauchy) and liquid (Lord Kelvin) continua. For slow quasi-stationary motions it behaves as a viscous fluid with viscosity being the cause of the Ohmic resistance. For fast oscillatory motions the viscoelastic liquid behaves essentially as an elastic continuum, especially as far as the shear waves (light) are concerned. This led the present author to conjecture [9,10] that the electromagnetic field is a viscoelastic liquid. It pervades the material Universe and can be called the *metacontinuum*. The brief outline of the continuum mechanics of the metacontinuum is presented bellow. Note that in the earlier works [7,8] only the elastic facet of the model was considered.

The Cauchy momentum balance equations and the continuity equation read

$$\mu\left[\frac{\partial \boldsymbol{v}}{\partial t}+\boldsymbol{v}\cdot\nabla\boldsymbol{v}\right]=-\nabla p+\nabla\cdot T=-\nabla p+\boldsymbol{t},\qquad \frac{\partial\mu}{\partial t}+\nabla\cdot(\mu\boldsymbol{v})=0, \tag{1}$$

where μ is the density, $\boldsymbol{u}$ is the displacement, $\boldsymbol{v}\equiv\partial\boldsymbol{u}/\partial t$ is the velocity, p is the pressure of the metacontinuum, and T is the deviator stress tensor. Respectively, $\boldsymbol{t}\equiv\nabla\cdot T$ is the effective body force to which the action of the internal stresses is reduced. It is directly related to the concept of stress vector and will be loosely called, in what follows, the "stress vector".

As shown in [10], when the bulk coefficient of viscosity is much larger than the shear coefficient, then one gets $\nabla\cdot\boldsymbol{v}=0$ and $\mu=$const, and hence the constitutive relation involves only the shear coefficient of viscosity, η, and the relaxation time, τ, namely

$$\tau\frac{DT}{Dt}+T=\eta D,\quad D=\frac{1}{2}[\nabla\boldsymbol{v}+\nabla\boldsymbol{v}^T], \tag{2}$$

where D is the rate of strain tensor and D/Dt denotes an appropriate invariant derivative (convective, upper/lower convected, etc.), which will be specified in what follows. One can write the constitutive law in terms of the stress vector (see [10]):

$$\tau\frac{D\boldsymbol{t}}{Dt}+\boldsymbol{t}=\eta\nabla\cdot D=\eta\Delta\boldsymbol{v}=-\eta\nabla\times\nabla\times\boldsymbol{v}\quad\text{for}\quad\nabla\cdot\boldsymbol{v}=0. \tag{3}$$

Eqs. (1) and (3) form a coupled system describing the motion of the viscoelastic metacontinuum. This system lends itself to a far-reaching analogy if one calls the negative stress vector the "electric force," and defines the "magnetic field," $\boldsymbol{H}$, as the vorticity in the metacontinuum, namely

$$\boldsymbol{E}\stackrel{\text{def}}{=}-\boldsymbol{t}=-\nabla\cdot T,\qquad \boldsymbol{H}\stackrel{\text{def}}{=}\nabla\times\boldsymbol{v}. \tag{4}$$

Upon introducing these in Eq. (1) and taking a *curl* of the latter, one obtains *Faraday's law*

$$\frac{\partial\boldsymbol{B}}{\partial t}+\boldsymbol{v}\cdot\nabla\boldsymbol{B}-\boldsymbol{B}\cdot\nabla\boldsymbol{v}-\nabla\times\boldsymbol{E}\Rightarrow\nabla\times[\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B}]=-\frac{\partial\boldsymbol{B}}{\partial t}, \tag{5}$$

with the Lorentz force already accounted for. As usually, $\boldsymbol{B}\stackrel{\text{def}}{=}\mu\boldsymbol{H}$ is called magnetic induction. Upon assuming that the stress in the *metacontinuum* produces a current $\boldsymbol{j}=\eta^{-1}\boldsymbol{E}$ the constitutive equation, Eq. (3), yields

$$\frac{D\boldsymbol{E}}{Dt}+\frac{\boldsymbol{j}}{\varepsilon}=c^2\nabla\times\boldsymbol{B}. \tag{6}$$

If the material derivative is reduced to the partial time derivative, Eq. (6) is nothing else but the second of Maxwell's equations, with $\varepsilon=\tau/\eta=1/(c^2\mu)$ playing the role of electric permittivity.

So far, we have shown that the Maxwell–Hertz equations are a direct corollary of the governing equations of the viscoelastic *metacontinuum*. The only undefined element at this point is the type of the invariant time derivative to be used. Since the model of any material must be *Frame Indifferent*, we proposed in [9] the Material Invariance Principle:

> Any formulation of electrodynamics, e.g., a constitutive relation such as Maxwell's displacement current, must have the same form in any coordinate system that moves and deforms with the continuum.

This principle must be used *in lieu* of the Lorentz Covariance (Relativity Principle). This requires that the Oldroyd upper-convected derivative [9]

$$\frac{D\boldsymbol{t}}{Dt} = \frac{\partial \boldsymbol{t}}{\partial t} + \boldsymbol{v}\cdot\nabla\boldsymbol{t} - \boldsymbol{t}\cdot\nabla\boldsymbol{v} + \boldsymbol{t}(\nabla\cdot\boldsymbol{v}), \tag{7}$$

should be used in the constitutive law, not merely the partial time derivative or convective derivative. For an incompressible medium the last term in Eq. (7) can be neglected. In terms of the electric and magnetic fields, the invariant formulation of the metacontinuum is summarized as

$$\frac{\partial \boldsymbol{B}}{\partial t} + \boldsymbol{v}\cdot\nabla\boldsymbol{B} - \boldsymbol{B}\cdot\nabla\boldsymbol{v} = -\nabla\times\boldsymbol{E}, \tag{8}$$

$$\frac{\partial \boldsymbol{E}}{\partial t} + \boldsymbol{v}\cdot\nabla\boldsymbol{E} - \boldsymbol{E}\cdot\nabla\boldsymbol{v} = c^2\nabla\times\boldsymbol{B} - \varepsilon^{-1}\boldsymbol{j}, \tag{9}$$

$$\nabla\cdot\boldsymbol{B} = 0. \tag{10}$$

Note, that in [9] the thrid term on the l.h.s. of Eq. (8) was erroneously omitted. The corrected version here shows that despite of the fact that the original equations contain the mere convective derivative for the velocity of the metacontinuum, now Eq. (8) and Eq. (9) both involve Oldroyd's upper convected derivative. This is an essential difference from Hertz's equations. In our formulation, the velocity and the pressure of the metacontinuum can be considered as a gauge to Eqs. (8) and (9) ("physical gauge").

The extended Maxwell electrodynamics outlined here is material frame indifferent. This gives the justification to call the results of the present section the "Special Theory of Absolutivity." In the limiting case of inertially moving non-deforming frames, this reduces to Galilean invariance.

3. Localized torsional dislocations and the absolute continuum: Lorentz contraction

In order to understand the interplay between the absolutivity of the metacontinuum and the relativity of the rectilinear motion, we consider a 2D case with only two nonzero displacement components components, $u_x(x,y)$ and $u_y(x,y)$. Also, we consider a case when there is no motion of the frame, i.e., there is no predominant velocity component. Then, assuming that the velocity components are small, one can linearize Eqs. (8) and (9) obtaining the classical Maxwell's equations.

For the 2D motion under consideration we can introduce a "displacement function" (similarly to the stream function in incompressible fluids, namely $u_x = \partial_y\psi$, $u_y = \partial_x\psi$). Then, from the linearized governing Eqs. (8) and (9) the following wave equation can be derived (if the conductivity is neglected):

$$\partial_{tt}\psi - c^2(\partial_{xx} + \partial_{yy})\psi = 0. \tag{11}$$

It is interesting to note that the last equation has a stationary solution with polar symmetry: $\psi = \ln(r)$, where $r = \sqrt{x^2+y^2}$, which is a localized solution for the component B_z of the magnetic induction. For the displacement components one has the following expressions $u_x = y/r^2$, $u_y = -x/r^2$. This solution is analogous to the well known potential vortex which has a nontrivial circulation (topological charge). This suggests that the localized torsional deformations can be interpreted as the charges. In order to distinguish the torsional elastic deformation from the fluid vortex, we call the fromar the 'gnarl.'

Now, we examine the situation when a localized solution of the above type *propagates* with phase velocity $\boldsymbol{v} = (v_1, v_2)$. Note that even for large phase speed, $|\boldsymbol{v}|\sim c$, the actual magnitudes of the velocities of the material particles of the metacontinuum are still very small and one can still use the linearized equation (Eq. (11)). Then, in a moving frame, $\xi = x - v_1 t$, $\eta = y - v_2 t$, one gets from Eq. (11) the following

$$(1 - v_1^2)\psi_{xx} + (1 - v_2^2)\psi_{yy} = 0 \Rightarrow \psi = \ln z, \quad z = \sqrt{x^2\left(\frac{1 - v_1^2}{c^2}\right)^{-1} + y^2\left(\frac{1 - v_2^2}{c^2}\right)^{-1}}. \tag{12}$$

The analytic form of the solution is the same, but for a different radial-like coordinate, z. The lines of constant z are ellipses, i.e., one is faced with a vortex whose steamlines are ellipses. This means that the moving phase pattern, Eq. (12), undergoes contraction in the direction of motion given exactly by the Lorentz factor. Since, the potential of interaction of two localized waves (see [11] for a similar derivation in the case of *sine*-Gordon equation) depends on the asymptotic behavior of their "tails". The fact that the charges are shortened in the direction of motions will lead to shortening of the distances between them in the same direction. This means that an assemblage of charges (a body) will be shortened in the direction of motion by the Lorentz factor.

Thus the relativity of rectilinear motion of phase patterns and their *Lorentz contraction* are manifestations of the *absolutivity of space*. The patterns *propagate over* rather than *move through* the *metacontinuum* and do not create an "aether wind", i.e., their translation does not contradict the principle of material invariance. One should be reminded here that light is the shear wave in the metacontinuum with propagation speed independent of the motion of the emitter.

4. The absolute field in higher dimensions (general absolutivity?)

Recall that the Schrödinger equation of wave mechanics is not one of Maxwell's equations (which points at the existence of another spatial dimension). As argued by Hinton [17], the thickness of the material world in the direction of the fourth dimension is so minute that it cannot be appreciated directly. Rather, it shows up through additional forces and/or variables. The most natural conjecture is that Schrödinger's wave function is such a manifestation of an underdeveloped fourth dimension. The question that arises here is the following: can one come up with a mechanical construct modeled by a fourth-order (generally nonlinear) dispersive equation that can provide the mechanical interpretation for Schrödinger's equation? The hint is in the original paper of Schrödinger [22] who observed that, in 2D, the equation for the real part of his wave function is the governing equation of a momentum-supporting material surface (an elastic plate or shell).

We begin the demonstration of this idea by considering the Schrödinger equation for the complex wave function ψ, which has the form

$$\boldsymbol{i}\frac{\partial \psi}{\partial t} + \hbar\Delta\psi - \chi\psi = 0, \quad \boldsymbol{i} \equiv \sqrt{-1}, \quad \psi \equiv \psi_1 + \boldsymbol{i}\psi_2, \tag{13}$$

where $\hbar$ is the reduced Planck constant and χ is construed to be connected to the potential of any external forces. The main characteristic of Eq. (13) is that it is not a wave equation, but contains dispersion which makes it qualitatively different from the models involving a wave equation (e.g., strings, membranes, superstrings, branes, etc.) Indeed, this can be easily seen if it is written in terms of the real (or imaginary) part of wave function, namely

$$\frac{\partial^2 \psi_1}{\partial t^2} + \hbar^2\Delta\Delta\psi_1 - 2\chi\hbar\Delta\psi_1 + \chi^2\psi_1 = 0. \tag{14}$$

For $\chi = 0$, Eq. (14) is nothing else but the Euler–Bernoulli equation for the flexural deformations of thin plates (as mentioned in [22]). Hence, we conjecture that the *metacontinuum* is a thin 3D material layer in 4D space. Thus, if the time is included, we arrive at a five dimensional model, but the new theory presented here is not related to the theory of Kaluza [18] and Klein [19]. While the model of Kalutza–Klein requires a Minkowski space-time continuum, the fourth spatial dimension of our work pertains to a material world with very clear mechanical interpretation, which does not require invoking the concept of probability.

Consider a very thin *momentum supporting* (visco)elastic structure of a 4D material. A 3D hypershell is the mathematical abstraction for this kind of momentum supporting material structure. Following the clues from the first part of the paper, we assume that the shear modulus is much smaller than the dilation one (i.e., the 4D material is virtually incompressible). In the middle 3-surface of the hypershell, the equations for the laminar components yield the governing equations of electrodynamics. The displacement, ζ, along the fourth dimension is the wave function. Contrary to the shell theory of technological applications, one has to consider the limiting case when the deflections are small, the strains (gradients) are of unit-order, and curvatures are large. Such an object is geometrically strongly nonlinear. The *hypershell* of this work is radically different form the notion of a *superstring* because the former can support momenta (the fourth order derivatives) while the latter cannot.

If h is the thickness of the shell, and L is the length scale of the localized deformation, we assume than $h \ll L \ll 1$. Then for the deflection, ζ, along the fourth dimension, the following governing equation can be derived [6] when the flexural deformations are not coupled to the deformations in the middle surface, namely

$$\mu \frac{\partial^2 \zeta}{\partial t^2} = \mu \mathcal{F} + G[-\Delta\Delta\zeta - (\Delta\zeta)^3] + \sigma\Delta\zeta, \tag{15}$$

where G is the stiffness of the shell, σ is the membrane tension, and $\mathcal{F}$ is a 4D body force. A dimensionless form

$$\frac{\partial^2 \zeta'}{\partial t^2} = \beta[-\Delta\Delta\zeta' - (\Delta\zeta')^3] + \Delta\zeta' + \mathcal{F}' \tag{16}$$

is obtained (note that $c_f \neq c$) by using the scales:

$$\zeta = L\zeta', \qquad \boldsymbol{x} = L\boldsymbol{x}', \qquad t = Lc_f^{-1}t', \qquad c_f^2 = \frac{|\sigma|}{\mu}, \qquad \beta = \frac{G}{|\sigma|L^2}. \tag{17}$$

Note that the linear part of (16) has the form proposed by Schrödinger if the membrane tension is understood to be related to the heuristic potential. Henceforth, the primes will be dropped and dimensionless variables are understood.

The shell–Universe can have the form of a sheet or a sphere with very large radius. In these two cases, the master equation does not contain the mean curvature of the Universe. The sheet/shell may be either compressed (negative membrane tension) or dilated (positive membrane tension). In both cases the basis state (the “vacuum state”) is the undisturbed surface and any other state should be a composition of localized disturbances. At this point it is too early to select a specific cosmological model. The case with negative mean membrane tension (bubble–Universe under external hydrostatic pressure from the 4D space) has been considered in [7]. Here we consider the case with positive mean membrane tension (the pressure acts from inside of the bubble.

Suppose that we are interested in an isolated system which occupies the domain B. Then at the boundary of the domain, $\boldsymbol{x} \in \partial B$, one has the trivial boundary conditions $\zeta = \Delta\zeta = 0$. It is interesting to note here that Eq. (16) does not admit a standard Hamiltonian formulation. Yet, for $F = 0$, after multiplying Eq. (16) by $\Delta\zeta$, we can get the following Hamiltonian density

$$H = \frac{1}{2}\left[\left(\frac{\partial\nabla\zeta}{\partial t}\right)^2 + (\Delta\zeta)^2 - \frac{1}{2}\beta(\Delta\zeta)^4 + \beta(\nabla\Delta\zeta)^2\right], \tag{18}$$

which indicates that the Lagrangian must have the form

$$L = \frac{1}{2}\left[\left(\frac{\partial\nabla\zeta}{\partial t}\right)^2 - (\Delta\zeta)^2 + \frac{1}{2}\beta(\Delta\zeta)^4 - \beta(\nabla\Delta\zeta)^2\right]. \tag{19}$$

Then, the Euler–Lagrange equation reads

$$\Delta\left[\frac{\partial^2\zeta}{\partial t^2} + \beta\Delta\Delta\zeta + \beta(\Delta\zeta)^3 - \Delta\zeta\right] = 0, \tag{20}$$

which is actually the Laplacian of the original equation, Eq. (16). In order to ensure that it has the same solutions, one has to enforce also the satisfaction of the original equation at the boundary ∂D. For asymptotic b.c., this requirement is trivially satisfied. Henceforth, we will use the Hamiltonian formulation, Eqs. (18) and (19).

The wave momentum and energy are defined as

$$\boldsymbol{P} \stackrel{\text{def}}{=} -\int_{\boldsymbol{B}} \zeta_t \nabla\Delta\zeta d^3\boldsymbol{x}, \qquad E \stackrel{\text{def}}{=} \int_{\boldsymbol{B}} H d^3\boldsymbol{x}, \tag{21}$$

respectively. The energy is conserved, i.e., $dE/dt = 0$, while for the wave momentum $\boldsymbol{P}$ a balance law holds

$$\frac{d\boldsymbol{P}}{dt} = \oint_{\partial B} H\boldsymbol{n}\, d\sigma \stackrel{\text{def}}{=} \mathcal{F}_p. \tag{22}$$

Note that $\boldsymbol{P}$ will be conserved too if the *pseudoforce* $\mathcal{F}_\mathrm{p} = 0$, which is also automatically satisfied for asymptotic b.c.

In concluding this section we can call the proposed concept "General Theory of Absolutivity", since to the generalized Maxwell dynamics of Section 2 (the "Special Theory of Absolutivity") it plays the same role as the General Relativity to Special Relativity. The main difference is that no space-time continuum is assumed and the presence of an additional variable is explained with the existence of an additional spatial dimension while the time is the fifth dimension.

5. Solitons and quasi-particles (Schrödinger's *Shaumkommen*?)

Russell [21] observed in 1834 a permanent wave (the "Great Wave") on the surface of a shallow-water layer (channel). In 1871–1872, Boussinesq [3] came up with a fundamental idea: dispersion balances the nonlinearity making the shape of the wave permanent. In other words, a profile can exist that propagates much in the same fashion as in the linear wave equation. The equation derived by Boussinesq spawned a variety of models, including wave propagation in elastic beams and shells (see, e.g., [14]).

Zabusky and Kruskal [24] discovered numerically in 1965 that solitary-wave solutions of *KdV* equation retain their shapes after multiple collisions. The coinage *soliton* was introduced by them to emphasize this kind of particle-like behavior. In the four decades that followed their work, important mathematical advances have been made in soliton theory. The fully integrable systems with infinite number of conservation laws were treated exhaustively by means of different analytical techniques: inverse scattering, Bäcklund transformation, Hirota bilinear technique (see, e.g., [1,4,20]).

From the physical point of view, the pertinent conservation laws are just three: mass, energy and momentum, i.e., the systems are rarely fully integrable. Energy-conserving schemes were developed, e.g., in the author's works, which emulated the conservation laws. Using adequate numerics, the shape-preserving nature of the interactions of QPs was established for the solutions of a variety of equations containing nonlinearity and dispersion. We computed numerically the interaction of two *sech* solitons of the 1D regularized wave equation, $u_{tt} = [u - u^2 - u_{tt}]_{xx}$ and showed that the solitary waves recover their original shapes after the collision despite of the non-fully-integrable nature of that equation [13]. This means that even in systems with just three conservation laws, the localized waves behave as quasi-particle (QPs), i.e., the full integrability of the system is not a necessary condition for the particle-like behavior of the solutions of type of localized waves.

The above described properties of QPs (for fully-integrable equations the latter are called solitons) mean that if one is faced with a nonlinear dispersive master equation, one can expect the appearance of solutions that are permanent localized structures, or QPs. This hints at the idea that the QPs of the master equation for the wave function are actually the particles. In other words, a "point particle" is our perception of a localized wave with very short support. In fact, since our "master equation" of wave mechanics is the nonlinear dispersive equation for the flexural deformations of a thin shell, then the center of a localized flexural deformation of the shell is perceived as a point particle. We can call with proper justification this kind of a localized wave the *flexons*.

In order to implement the above formulated concept, we need to first find a localized solution of the "master" equation. Unfortunately, the 1D case cannot serve as a simplified "laboratory model" from which one can infer the shape of a QP in higher dimensions because the solution in 1D can have completely different behavior at infinity from the 3D solution. This means that we have to focus our attention on an essentially 3D case. The simplest such case is presented by the spherically-symmetric, stationary solution for which the master equation of wave mechanics, Eq. (16), can be recast as

$$\beta \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dw}{dr}\right) - w + w^3 = 0, \qquad w(r) = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\zeta}{dr}\right), \tag{23}$$

where w is the curvature of the flexural deformation.

We begin with computing the localized solution of Eq. $(23)_1$. In 1D, the respective equation has the famous *sech* solution, but in the spherical coordinates, no analytical solution is available. The difficulties for the numerical solution are connected with the infinite interval, and with the bifurcate nature of the non-trivial solution. For dealing with this kind of difficulties, we proposed the so-called Method of Variational Imbedding (see [12] for details and application to localized solutions). The case with negative membrane tension [7] was treated using MVI, and it was found that the solitary wave possess oscillatory tails decaying at infinity in a non-monotone fashion. After we find the solution of Eq.

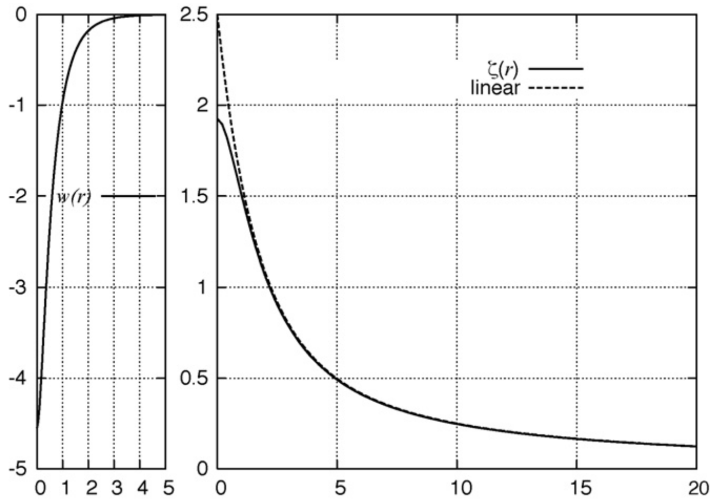

Fig. 1. Portrait of a *flexon*. *Left panel*: numerical solution for $w(r) \equiv \Delta\zeta(r)$. *Right panel*: Solution for ζ obtained with numerical quadrature of w (solid line) as compared to the properly scaled solution of the linearized equation (dashed line).

(23) for $w(r)$ we find the best fit to w, of the following type

$$w(r) = -0.484(2e^{-1.125r} + 16e^{-2.25r} - 16e^{-3.375r} + 23e^{-4.5r} - 15.6e^{-5.6125r}), \tag{24}$$

which is presented in the left panel of Fig. 1.

The deviation of the best-fit formula from the actually computed finite-difference solution is less than 0.3%. Using the best-fit approximation is crucial for the next step of the solution when the actual wave function, ζ, is defined. From Eq. (24), we find ζ analytically by means of repeated quadratures

$$\zeta(r) = \int_0^r \left[\frac{1}{r_1^2}\left(\int_0^r r_2^2 w(r_2) dr_2\right)\right] dr_1 = \zeta(r) = -0.23970e^{-5.6125r} + 0.54973e^{-4.5r} - 0.67986e^{-3.375r} + 1.5297e^{-2.25r} + 0.76484e^{-1.125r} + \frac{1}{r}(-0.085414e^{-5.6125r} + 0.24433e^{-4.5r} - 0.40288e^{-3.375r} + 1.3597e^{-2.25r} + 1.3597e^{-1.125r} - 2.4755), \tag{25}$$

which is presented in the right panel of Fig. 1 alongside the solution

$$\hat{\zeta}(r) = \frac{\alpha}{r}(1 - e^{-r}), \quad \alpha = \text{const}, \tag{26}$$

of the linearized version of Eq. (23) with $\alpha = 2.5$.

Thus we have arrived at the notion of a pseudo-localized solution, which itself does not have an integrable square over the infinite domain, but its curvature (the Laplacian of the profile) does. Note that such solutions may or may not decay at infinity, but the Laplacian of the solution decays fast enough in order to have integrable square in the infinite domain.

6. Charged quasi-particles

In a future work, the author intends to derive the equation of the flexural deformation (the wave mechanics) in the next asymptotic order which will include the interaction with the laminar 3D components (electrodynamics). It is rather technical (see, e.g., [15]) and goes beyond the scope of the present paper, the latter being concerned mostly with general outline of the unified field theory. The displacement field of a charged QP is shown in Fig. 2 for the 2D case for illustrative purposes.

The vortex structure now is in the middle surface of the shell, rather than on a flat surface. When the respective equations are derived, then one will be able to investigate the co-centered shear solitons (charges) with flexural solitons (particles), i.e one will be able to look deeper into the idea of charged QPs.

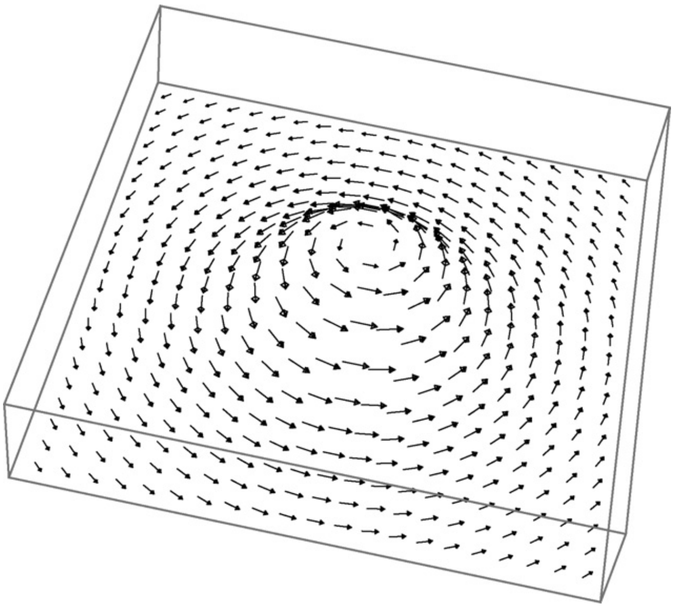

Fig. 2. A 2D portrait of a charged quasi-particle: the velocity field in the bended middle surface of the shell.

7. Wave particle syncretism (non-probabilistic wave mechanics?)

Let us begin with an isolated localized wave of shape $F(\boldsymbol{x})$ that propagates along a path $\boldsymbol{X}(t)$ without changing shape. Then

$$\zeta = F[\boldsymbol{x} - \boldsymbol{X}(t)] + U(\boldsymbol{x}) \Rightarrow \frac{\partial \nabla \zeta}{\partial t} = (\nabla\nabla F) \cdot \dot{\boldsymbol{X}},$$

where $U(\boldsymbol{x})$ is a background deformation (a "potential"). For the system consisting of a single object with trajectory $\boldsymbol{x} = \boldsymbol{X}(t)$, one can derive the discrete Lagrangian

$$\mathfrak{L} = \frac{1}{2}\mathfrak{M}\dot{X}\dot{X} - \mathfrak{U}(X), \quad \text{where} \quad \mathfrak{M} \stackrel{\text{def}}{=} \int_B (\nabla\nabla F) \cdot (\nabla\nabla F) d^3 x, \tag{27}$$

is the tensor of *pseudomass* ("mass of curvature") and

$$\mathfrak{U}(\boldsymbol{X}) \stackrel{\text{def}}{=} \int_B \left\{ \Delta F(\boldsymbol{x} - \boldsymbol{X}) \Delta U(\boldsymbol{x}) + \beta \nabla \Delta F(\boldsymbol{x} - \boldsymbol{X}) \cdot \nabla \Delta U(\boldsymbol{x}) - \frac{3}{2}\beta \Delta F(\boldsymbol{x} - \boldsymbol{X}) \Delta U(\boldsymbol{x}) [\Delta F(\boldsymbol{x} - \boldsymbol{X}) + \Delta U(\boldsymbol{x})] \right\} d^3 \boldsymbol{x}$$

is the potential defined up to an arbitrary constant. Note that the domain B is considered to be very large and for the purposes of the present work the integrals can be taken over the infinite domain. The QPs from the previous section possess spherical symmetry, i.e., $F = F(r)$, $r = |\boldsymbol{x}|$. Then, for a single QP, the tensor of *pseudomasses* is reduced to a single scalar coefficient, namely

$$\mathfrak{L} = \frac{1}{2}\mathfrak{m}\dot{\boldsymbol{X}}^2 - \mathfrak{U}(\boldsymbol{X}), \quad \text{where} \quad \mathfrak{m} = \int_0^\infty \left[\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dF}{dr} \right) \right]^2 r^2 dr. \tag{28}$$

Note that for the previously shown numerically computed shape we get $\mathfrak{m} = 1.47204$.

Under the assumption that the motion of the QP does not have impact on its shape, we get for the momentum the following balance law

$$\frac{d}{dt}\boldsymbol{P} = \frac{d}{dt}[\mathfrak{m}\dot{\boldsymbol{X}}] = \nabla_X \mathfrak{U}, \tag{29}$$

which is the second Newton law for a particle moving in an external potential. Thus, the discrete Lagrangian, Eq. (28) provides that if the shape of the QP does not change during the motion, then it is a Newtonian particle. Any change of the shape under the action of the external forces or for other dynamical reasons will make the QP behave like a non-Newtonian particle.

Some call the above approach the "method of collective variables" [23,5], which is, in turn a "variational approximation" [2]. As pointed out in [11] one can obtain a "coarse-grain description" of the continuous field by assuming that the solution is a superposition of the fields created by the presence of each QP. This leads to the application of the "coarse-grain" description to the interaction of two quasi-particles which was thoroughly tested for the case of *sine*-Gordon equation in [11] for the case of two QPs whose shapes are not influenced by their interaction. We apply the same idea to the pseudo-localized waves of the previous section. For two quasi-particles with shapes $F^{(i)}$ and trajectories $\boldsymbol{X}_i(t)$, we have

$$\zeta = F^{(1)}[\boldsymbol{x} - \boldsymbol{X}_1(t)] + F^{(2)}[\boldsymbol{x} - \boldsymbol{X}_2(t)] + F^{(12)}[\boldsymbol{x} - \boldsymbol{X}_1(t), \boldsymbol{x} - \boldsymbol{X}_2(t)]. \tag{30}$$

If one is interested in the case when the localized structures are not interacting significantly (i.e., they are well separated), then $F^{(12)}$ can be neglected. Such an approach is consistent with the notion of a particle, in the sense that the distance between two structures is larger than the size of each of the structures. If one is to consider the dynamics of significantly overlapping structures ("cross-section" of collision of two QPs) one has to solve the PDE for the flexural deformations of the hypershell, Eq. (16). Yet, it is important to bear in mind that at reasonable distances from each other, the localized structures do indeed have particle-like behavior and the term $F^{(12)}$ in Eq. (30) can be neglected. Then

$$\frac{\partial \nabla \zeta}{\partial t} = -\frac{d\boldsymbol{X}_1}{dt} \cdot \nabla\nabla F^{(1)} - \frac{d\boldsymbol{X}_2}{dt} \cdot \nabla\nabla F^{(2)} \tag{31}$$

and the term $(\nabla \zeta_t)^2$ in the Lagrangian yields

$$\mathfrak{L} = \sum_{i=1}^{2}\sum_{j=1}^{2} \mathfrak{M}_{ij} \dot{\boldsymbol{X}}_i \dot{\boldsymbol{X}}_j - \mathfrak{U}(\boldsymbol{X}_2 - \boldsymbol{X}_1), \quad \mathfrak{M}_{ij} = \int \nabla\nabla F^{(i)}(|\boldsymbol{x} - \boldsymbol{X}_i|) : \nabla\nabla F^{(j)}(|\boldsymbol{x} - \boldsymbol{X}_j|) d^3\boldsymbol{x}. \tag{32}$$

For the case of spherical symmetry $\mathfrak{M}_{22}, \mathfrak{M}_{11}$ are spherical tensors, i.e., each of them is reduced to the mass of a single QP,. For the above described *flexon*, $\mathfrak{m}_{22} = \mathfrak{m}_{11} = 1.47204$. At this stage we were unable to prove that the tensor of the "cross-masses", $\mathfrak{M}_{12}$, is spherical. What can be argued *ad hoc*, however, is that $\mathfrak{M}_{12}(\boldsymbol{x}) \sim |z|^{-1}$ for $|z| = |\boldsymbol{X}_2 - \boldsymbol{X}_1| \gg 1$.

Now, for the potential one gets (denote $G(\boldsymbol{x}) \equiv \Delta F(\boldsymbol{x})$):

$$\mathfrak{U}(z) \stackrel{\text{def}}{=} \int_D \Big[G(\boldsymbol{x} - \boldsymbol{X}_1) G(\boldsymbol{x} - \boldsymbol{X}_2) - \frac{3}{2}\beta G(\boldsymbol{x} - \boldsymbol{X}_1) G(\boldsymbol{x} - \boldsymbol{X}_2)\{\Delta G(\boldsymbol{x} - \boldsymbol{X}_1) + G(\boldsymbol{x} - \boldsymbol{X}_2)\}$$
$$+ \beta \nabla G(\boldsymbol{x} - \boldsymbol{X}_1) \cdot \nabla G(\boldsymbol{x} - \boldsymbol{X}_2)\Big] d^3\boldsymbol{x} + \text{const},$$

and the Euler–Lagrange equations for the coarse-grain description give the generalization of Newton's second law:

$$\begin{aligned} &\frac{d}{dt}\mathfrak{m}_{11}(z)\dot{\boldsymbol{X}}_1 + \frac{d}{dt}[\mathfrak{M}_{12}(z) \cdot \dot{\boldsymbol{X}}_2] = -\nabla \mathfrak{U}(z), \\ &\frac{d}{dt}[\mathfrak{M}_{21}(z) \cdot \dot{\boldsymbol{X}}_1] + \frac{d}{dt}\mathfrak{m}_{22}(z)\dot{\boldsymbol{X}}_2 = \nabla \mathfrak{U}(z). \end{aligned} \tag{33}$$

Some preliminary rough estimates put the asymptotic behavior of $\mathfrak{U} \sim |z|^{-1} + \text{const}$ for $|z| \gg 1$. Respectively $\nabla \mathfrak{U} \sim |z|^{-2}$, which means that an attractive force acts upon the centers of particles proportional to the inverse of the distance square. The rigorous proof of this is rather lengthy and involves expansions of triple integrals with respect to the parameter **z** and will be done elsewhere. We just mention, that the law of attraction at large distances reflects the fact that one is faced with *pseudo*-localized solution. Note that for truly localized shapes (such as the curvature w or the *sech*-es in 1D), the force of interaction diminishes exponentially with the distance between the QPs.

The concept put forward in the present section allows us to reconsider the so-called Wave-Particle duality. If unaware of the absolute hypershell (the metacontinuum) one may try to explain the presence of the wave function (the amplitude of the flexural deformations) as an "associated" wave function and may endow it, e.g., with probabilistic meaning. The intuitive basis for the probabilistic interpretation is rooted in the fact that the local amplitude of the wave is what is detected as the main manifestation of the localized wave (the QP) and it is natural to come up with the concept that in some points one has "more particle" (higher amplitude of the flexural wave) and in some other places one sees "less particle". It is easy to envision this undulations of the "corpuscularness" as a probability to find a particle in the specific geometric point. The present works shows that if one understands the nature of the absolute medium (the metacontinuum) one comes up with a much more natural (even prosaic) interpretation of the wave function as the amplitude of the localized undulations of the hypershell. This is a clear non-probabilistic interpretation of wave mechanics. (Actually, it is a *mechanical* theory of wave mechanics.) The quantum nature of the localized waves stems from the fact that they are solutions of nonlinear eigen-value problems of the nonlinear "master" wave equation proposed in the present work.

8. Conclusions: the unified field theory

In this paper we have shown how the model of a viscoelastic hypershell (called the *metacontinuum*) can unify electromagnetism, gravitation, and Schrödinger's wave mechanics, as well as explain the Wave-Particle duality. The metacontinuum of our work is not the classical aether because it is a very dense viscoelastic medium. It was introduced in earlier works of the present author in an attempt to provide a mechanical model of electrodynamics.

The notion of an absolute medium is extended here to assume that the *metacontinuum* is a 3D hypershell in the 4D real (non-Minkowskian) space for which time is the fifth independent variable. A nonlinear equation of Boussinesq type ("master" equation of wave mechanics) governs the flexural deformations of the shell. Its linear part gives the Schrödinger equation when written for the real or imaginary part of the wave function.

The central element of the new concept is the assumption that the particles are localized waves of the governing equations of metacontinuum. The Hamiltonian formulation of the metadynamics defines the Newtonian dynamics of the centers of the localized waves. They do not move through the metacontinuum (as believed both in the 19th century aether theories and in the modern field theories), rather they *propagate* over its surface as phase patterns (quasi-particles or QPs). This leads to the notion of syncretism between waves and particles rather than duality.

In the middle surface of the *hypershell*, torsion localized waves with quasi-particle behavior can exist which are interpreted as charges (quasi charges or QCs). The linear shear waves (light) propagate with absolute phase speed irrespective to the relative motion of the QCs, while the latter experience contraction in the direction of *propagation* proportional to the Lorentz factor.

According to the concept formulated in the present paper, the Universe is a fusion between continuous and discrete: an absolute continuum where light propagates and localized phase patterns in relative motion, which patterns are perceived as particles.

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